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Instability of Dc-Dc Converters at the Boundary Between CCM and Discontinuous Capacitor Voltage Mode

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Abstract – In this paper, the operating mode between Continuous Conduction Mode (CCM) and Discontinuous Capacitor Voltage Mode (DCVM) of dc-dc converters is investigated. A large-signal model for the switching network is obtained. The small-signal averaged model is derived, and it is used to obtain analytical expressions for the small-signal transfer functions, for the ĆUK, SEPIC and ZETA converters. It is shown that the transfer functions exhibit at least one right half-plane (RHP) pole. This pole cannot be eliminated by varying the circuit parameters. From this it is concluded that the operating mode between CCM and DCVM is unstable and unusable.

Keywords: boundary conduction mode, discontinuous capacitor voltage mode, averaged switch models.

I. INTRODUCTION

Occurrence of discontinuous modes in PWM dc-dc converters can be easily explained taking into account some topological aspects. In [1] it is shown that the transistor and the diode in every PWM converter form:

- a loop `L`, with possibly the supply voltage, V_g, and a (possibly empty) set of capacitors;
- a cut-set `C`, with a non-empty set of inductors.

In DCVM the small ripple assumption is invalid for at least one capacitor in the loop `L`. A necessary and sufficient condition for the occurrence of DCVM is the existence of at least one capacitor in the loop `L`, obviously different from the output filter capacitor. Therefore it is clear that DCVM cannot be related to BUCK, BOOST or BUCK-BOOST converters, as these converters contain a single capacitor for filtering the output voltage. On the other side, ĆUK, SEPIC and ZETA converters can enter DCVM mode when the small ripple assumption is removed from the energy storage capacitor contained in the loop `L`.

The large-signal averaged switch model of the switch network in pulse width modulated (PWM) dc-dc converters operating at the boundary between CCM and DCVM is derived in Section 2. In Section 3 the small-signal averaged switch model is derived and it is used to obtain analytical expressions for the small-signal transfer functions. A discution, based on the small-signal transfer functions, regarding the stability of this operating mode is also presented. In Section 4 the instability is verified through CASPOC simulation. Conclusions are presented in Section 5.

II. AVERAGED SWITCH MODEL FOR BOUNDARY CONDUCTION MODE BETWEEN CCM AND DCVM

Let us examine the operation at the Boundary Conduction Mode between CCM and DCVM of the ĆUK converter, and follow the averaged switch modelling approach [2] to derive an equivalent circuit that models the averaged (over one switching period, T_s) terminal variables of the switch network.

As it is known, for the inductor currents, i_{L1} and i_{L2} the negligible ripple assumption is still valid, and therefore, they can be admitted constant during one switching cycle. However, for the capacitor voltage, v_{C1} the small ripple assumption is not valid, while v_{C2} is assumed to be constant as it is in fact the output voltage.

The terminal variables of the switch network, i_1, i_2, v_1 and v_2 are defined in Fig. 1.

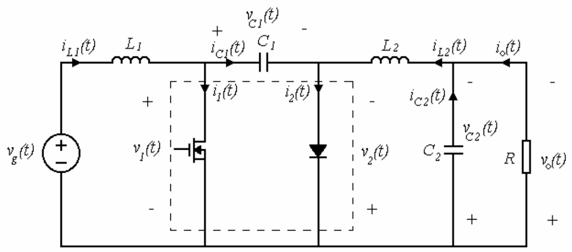

Fig. 1. The CUK converter, with the switch network identified.

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The switch network voltage and current waveforms are presented in Fig. 2.

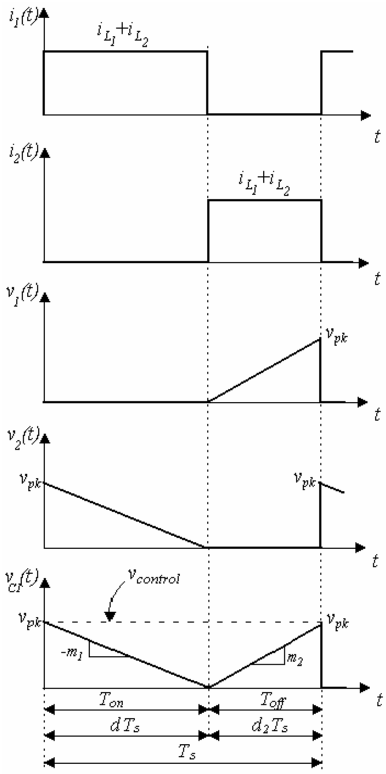

Fig. 2. The switch network and capacitor C_1 waveforms.

The peak value of the capacitor voltage, v_{pk} is equal to the control input, $v_{control}$. During the transistor on-time, in the first subinterval, the capacitor voltage decreases from the peak value with a slope $m_1 = \frac{i_{L2}}{C_1}$ until it reaches zero. At this time, the transistor is turned off, and the diode starts to conduct. In the second subinterval, during the diode on-time, the capacitor voltage increases from zero with a slope $m_2 = \frac{i_{L1}}{C_1}$ to the peak value. Therefore, the averaged value over one switching period, T_s, of the capacitor voltage, v_{C1} is given by:

$$< v_{C1} >= \frac{1}{2} < v_{pk} >= \frac{1}{2} < v_{control} > \qquad (1)$$

By averaging the waveforms in Fig. 2 over one switching period, T_s, and taking Eqn. (1) into account, the averaged switch network variables are found as:

$$< i_1(t) >= \frac{T_{on}}{T_s} (< i_{L1} > + < i_{L2} >) \qquad (2)$$

$$< i_2(t) >= \frac{T_{off}}{T_s} (< i_{L1} > + < i_{L2} >) \qquad (3)$$

$$< v_1(t) >= \frac{T_{off}}{T_s} \frac{< v_{control} >}{2} \qquad (4)$$

$$< v_2(t) >= \frac{T_{on}}{T_s} \frac{< v_{control} >}{2} \qquad (5)$$

From Eqns. (2)-(5) the relations between the switch network variables can be written as:

$$< v_1(t) >= \frac{T_{off}}{T_{on}} < v_2(t) > \qquad (6)$$

$$< i_2(t) >= \frac{T_{off}}{T_{on}} < i_1(t) > \qquad (7)$$

From Eqns. (6) and (7) it is obvious that:

$$< i_1(t) >< v_1(t) >=< i_2(t) >< v_2(t) >=< p(t) > \quad (8)$$

which is expected, as we considered ideal and therefore lossless active and pasive switches. In fact, Eqn. (8) denotes instantaneous power conservation in the switch network.

Based on Eqn. \ref{av5} and \ref{av6} the averaged large-signal switch model can be constructed, as in Fig. 3.

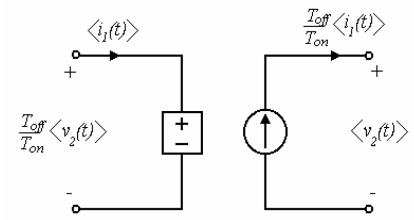

Fig. 3. The boundary mode averaged large-signal model.

The averaged switch models of the ĆUK, SEPIC and ZETA converters are shown in Fig. 4.

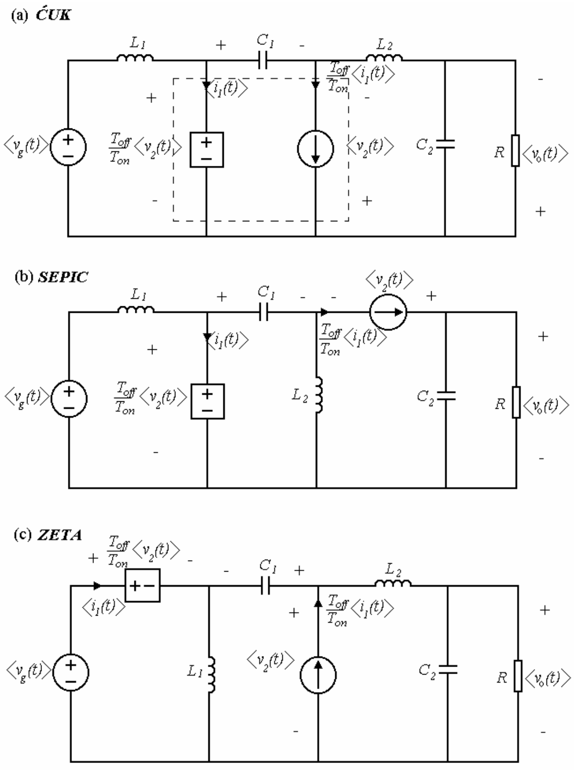

Fig. 4. Averaged large-signal equivalent circuits of the ĆUK, SEPIC and ZETA converters operating in boundary mode.

The steady-state average value for any converter are found by setting all averaged waveforms to their quiescent values and letting the inductors and capacitors become a short-circuit and an open circuit respectively. From Fig. 4 it can be observed that the dc conversion ratio, M, is equal to:

$$M = \frac{V_o}{V_g} = \frac{V_2}{V_1} = \frac{I_1}{I_2} \tag{9}$$

From Fig. 2 we have:

$$D = \frac{T_{on}}{T_s} \tag{10}$$

and

$$D_1 = 1 - D = \frac{T_{off}}{T_s} \tag{11}$$

where D is the steady-state duty cycle.

Using the Eqn. (6), (10) and (11), from Eqn. (9) we get:

$$M = \frac{D}{1-D} \tag{12}$$

III. SMALL-SIGNAL MODEL FOR BOUNDARY CONDUCTION MODE BETWEEN CCM AND DCVM

Small-signal models are constructed by perturbation and linearization of the large-signal averaged expressions. Let:

$$\begin{aligned} v_1(t) &= V_1 + \hat{v}_1 \\ i_1(t) &= I_1 + \hat{i}_1 \\ v_2(t) &= V_2 + \hat{v}_2 \\ i_2(t) &= I_2 + \hat{i}_2 \\ v_{control}(t) &= V_{control} + \hat{v}_{control} \end{aligned} \tag{13}$$

Perturbation of the Eqn. (8) leads to:

$$\left(I_1 + \hat{i}_1\right)\left(V_1 + \hat{v}_1\right) = \left(I_2 + \hat{i}_2\right)\left(V_2 + \hat{v}_2\right) \tag{14}$$

The dc terms of both sides of this equation are equal from the steady-state operation:

$$V_1 I_1 = V_2 I_2 \tag{15}$$

Taking Eqn. (11) into account and neglecting the product of two small perturbations, Eqn. (10) becomes:

$$I_1 \hat{v}_1 + \hat{i}_1 V_1 = I_2 \hat{v}_2 + \hat{i}_2 V_2 \tag{16}$$

From Eqn. (12) the solution for the small-signal switch network output voltage, $\hat{v}_2$, is given by:

$$\hat{v}_2 = \frac{I_1}{I_2}\hat{v}_1 + \frac{V_1}{I_2}\hat{i}_1 - \frac{V_2}{I_2}\hat{i}_2 \tag{17}$$

From the Eqn. (4) and (5) it can be observed that

$$< v_1(t) > + < v_2(t) > = \frac{1}{2} < v_{control} > \tag{18}$$

Perturbating Eqn. (18) we get:

$$\hat{v}_1 + \hat{v}_2 = \frac{1}{2}\hat{v}_{control} \tag{19}$$

In Eqn. (17) $\hat{v}_1$ is replaced from Eqn. (19), which gives:

$$\hat{v}_2 = \left(\frac{1}{1 + \frac{I_1}{I_2}}\right)\left(\frac{I_1}{2I_2}\hat{v}_{control} + \frac{V_1}{I_2}\hat{i}_1 - \frac{V_2}{I_2}\hat{i}_2\right) \tag{20}$$

From Eqn. (19) and (20) the small-signal switch network input voltage $\hat{v}_1$ is given by:

$$\hat{v}_1 = \left(\frac{1}{1 + \frac{I_1}{I_2}}\right)\left(\frac{1}{2}\hat{v}_{control} - \frac{V_1}{I_2}\hat{i}_1 + \frac{V_2}{I_2}\hat{i}_2\right) \tag{21}$$

By using the dc relations $V_2 = MV_1$, $I_1 = MI_2$, $I_2 = \frac{V_2}{R}$, Eqn. (20) and (21) become:

$$\begin{cases} \hat{v}_1 = \dfrac{1}{2(M+1)}\hat{v}_{control} - \dfrac{R}{M(M+1)}\hat{i}_1 + \dfrac{R}{M+1}\hat{i}_2 \\ \hat{v}_2 = \dfrac{M}{2(M+1)}\hat{v}_{control} + \dfrac{R}{M(M+1)}\hat{i}_1 - \dfrac{R}{M+1}\hat{i}_2 \end{cases} \tag{22}$$

Based on Eqn. (22) the small-signal switch model of the switch network can be constructed, as in Fig. 5.

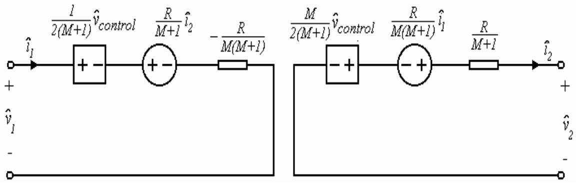

Fig. 5. The boundary mode small-signal model of the switch network.

The expressions of the small-signal model parameters are the same for both the ĆUK and for the SEPIC and ZETA converters.

The small-signal circuit model can be obtained by replacing the switch network with the two-port network described by Eqn. (22), as shown in Fig. 6, for the ĆUK, SEPIC and ZETA converters.

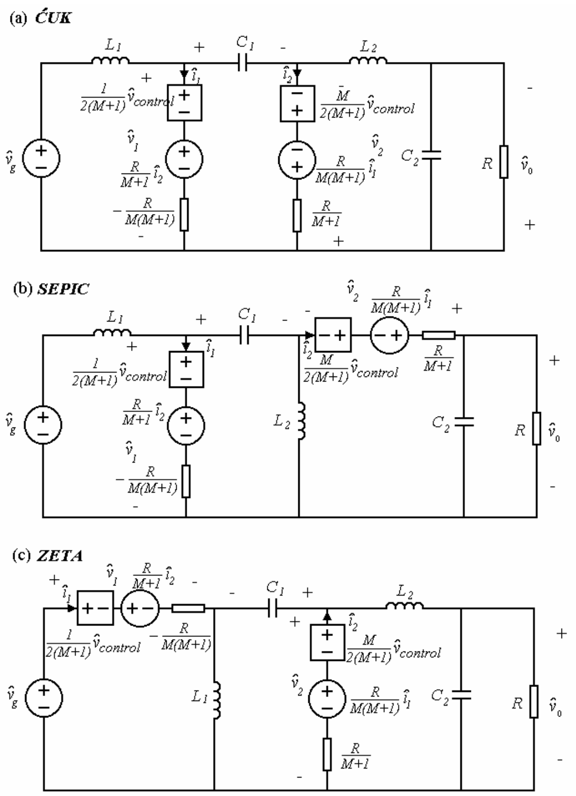

Fig. 6. Small-signal models of the ĆUK, SEPIC and ZETA converters.

By solving the small-signal equivalent circuits, the transfer functions can be found. For the ĆUK, SEPIC and ZETA converters, the transfer functions are given in Table 1.

It can be seen that the denominators of all transfer functions from Table 1 is a cubic polynomial. Its roots can be all three real, or one real and two compound numbers. In both cases, the free term is equal to the product of the roots, with the sign changed, which can be verified through a simple calculation. Therefore, if the free term has the opposite sign as against the s^3 term, as in the case of the transfer functions from Table 1, it results that all three roots are positive or the real root is positive respectively. It also can be observed that the sign of the free term does not depend on the circuit parameters. This means that the small-signal transfer functions exhibit at least one right half-plane (RHP) pole. This pole cannot be eliminated by varying the circuit parameters. From this it is concluded that the operating mode between CCM and DCVM is unstable, therefore unusable.

IV. SIMULATION RESULTS

In order to verify the results, a ĆUK converter and a SEPIC converter are examined. The component values are the following:

1. ĆUK converter: V_g = 20V, L_1 = 0,64mH, L_2 = 0,64m, R = 10Ω, C_1 = 90nF, C_2 = 800µF; V_o = 5V;
2. SEPIC converter: V_g = 15V, L_1 = 0,64mH, L_2 = 0,64m, R = 10Ω, C_1 = 56.8nF, C_2 = 100µF; V_o = 3.75V.

The poles of the transfer functions are, for the ĆUK converter: 37.517k, -71±1612i, and for the SEPIC converter: 37.633k, -566±4521i.

The capacitor voltage waveform for the ĆUK

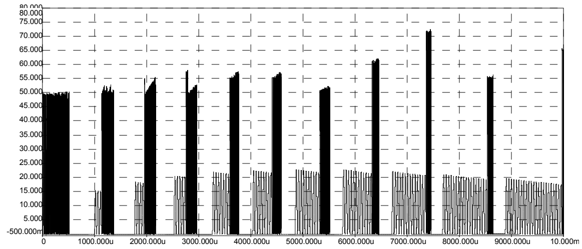

Fig. 7. The capacitor voltage waveform for the ĆUK converter.

The output voltage waveforms are shown in Fig. 8 (for the ĆUK converter) and in Fig. 9 (for the SEPIC converter). It can be observed the instability of this operating mode.

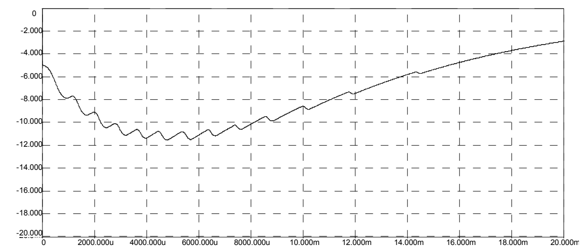

Fig. 8. The output voltage waveform for the ĆUK converter.

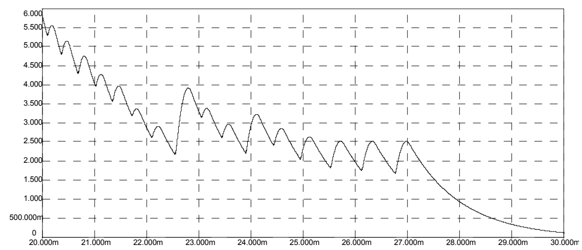

Fig. 9. The output voltage waveform for the SEPIC converter.

Table 1

Small-signal transfer functions
ĆUK
$G_{vc}(s)=\dfrac{\hat{v}_o}{\hat{v}_{control}}=\dfrac{\frac{1}{2}\left[1-sL_1\frac{M^2}{R}+s^2L_1C_1(M+1)\right]}{1-s\left[\frac{L_1M(M+2)}{R}-\frac{L_2}{R}\right]+s^2\left[-ML_1C_2+L_2C_2-\frac{L_1L_2M(M+1)}{R^2}\right]-s^3\frac{L_1L_2C_2M(M+1)}{R}}$
$G_{vg}(s)=\dfrac{\hat{v}_o}{\hat{v}_g}=\dfrac{1}{1-s\left[\frac{L_1M(M+2)}{R}-\frac{L_2}{R}\right]+s^2\left[-ML_1C_2+L_2C_2-\frac{L_1L_2M(M+1)}{R^2}\right]-s^3\frac{L_1L_2C_2M(M+1)}{R}}$
SEPIC
$G_{vc}(s)=\dfrac{\hat{v}_o}{\hat{v}_{control}}=\dfrac{\frac{1}{2}\left[1-sL_1\frac{M^2}{R}+s^2C_1(L_1+L_2)-s^3\frac{L_1L_2C_1M(M+1)}{R}\right]}{1-s\left[\frac{L_1M(M+2)}{R}-\frac{L_2}{R}\right]+s^2\left[-ML_1C_2+L_2(C_1+C_2)+L_2C_2-\frac{L_1L_2M(M+1)}{R^2}\right]-s^3\frac{L_1L_2(C_1+C_2)M(M+1)}{R}}$
$G_{vg}(s)=\dfrac{\hat{v}_o}{\hat{v}_g}=\dfrac{1}{1-s\left[\frac{L_1M(M+2)}{R}-\frac{L_2}{R}\right]+s^2\left[-ML_1C_2+L_2(C_1+C_2)+L_2C_2-\frac{L_1L_2M(M+1)}{R^2}\right]-s^3\frac{L_1L_2(C_1+C_2)M(M+1)}{R}}$
ZETA
$G_{vc}(s)=\dfrac{\hat{v}_o}{\hat{v}_{control}}=\dfrac{\frac{1}{2}\left[1-sL_1\frac{M^2}{R}+s^2L_1C_1(M+1)\right]}{1-s\left[\frac{L_1M(M+2)}{R}-\frac{L_2}{R}\right]+s^2\left[-ML_1C_2+L_2C_2-\frac{L_1L_2M(M+1)}{R^2}\right]-s^3\frac{L_1L_2C_2M(M+1)}{R}}$
$G_{vg}(s)=\dfrac{\hat{v}_o}{\hat{v}_g}=\dfrac{1+s^2L_1C_1(M+1)}{1-s\left[\frac{L_1M(M+2)}{R}-\frac{L_2}{R}\right]+s^2\left[-ML_1C_2+L_2C_2-\frac{L_1L_2M(M+1)}{R^2}\right]-s^3\frac{L_1L_2C_2M(M+1)}{R}}$

V. CONCLUSIONS

The operating mode between Continuous Conduction Mode (CCM) and Discontinuous Capacitor Voltage Mode (DCVM) of dc-dc converters is investigated. A large-signal model for the switching network of the pulse width modulated (PWM) dc-dc converters operating at the boundary CCM and DCVM is developed. The small-signal averaged model is derived, and it is used to obtain analytical expressions for the small-signal transfer functions, for the ĆUK, SEPIC and ZETA converters. It is shown that the transfer functions exhibit at least one right half-plane (RHP) pole. The RHP pole cannot be eliminated by varying the circuit parameters. From this it is concluded that the operating mode between CCM and DCVM is unstable and unusable. CASPOC simulations of the The ĆUK and the SEPIC converters are performed. The results confirm the instability of the operating mode between CCM and DCVM.

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