

## Interpolation of linear track movements of modern industrial robots

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**Abstract – Integration of linear track movements in the robot control is a significant part of the accuracy improvement process of modern robotics. For this, the linear track profile has to be analysed for getting a continous description of its inaccuracies for correcting the robot's end effector position for arbitrary points. This analysis bases in the first step on a discrete measurement of the linear track in special sampling points. In the second step an interpolation between the sampling points is done. In this article some of the possible interpolation methods for creating a continous description of the linear track were tested.**

**Keywords:** industrial robot, accuracy, 7<sup>th</sup> axis, linear track, interpolation

### I. INTRODUCTION

Modern robotics today deals with a great number of different applications. A great number of special functions or tasks are done by industrial robots. The field of using industrial robots ranges from picking or placing very small parts for electrical layouts to the painting or welding of jumbo jets. In each one of this varying types the demands on the robot are different and depending on single aspects like accuracy, time consumption or the costs.

One important fact is that in the past few years the demands of modern robotics have increased due to the ever growing demand for flexible automation. Flexible automation today expects highly accurate robotic systems. In this meaning not only the robot, but the complete system consisting of robot and its peripherals have to fulfill the extensive requirements. But against this demand for highly accurate robotic systems the requested flexibility in the used systems is increasing rapidly.

One example of the situation discussed in this document is the demand for manipulating very large workpieces which even the workspace for a huge robot is too small. To extend the workspace the robot is placed on a linear track, even called 7<sup>th</sup> axis, to move the robot linear in one determined direction. Using this extension practically any workpieces can be handled by robots suitable for the application

without the necessity of using large robots which have a sufficient workspace.

On the one hand the usage of the linear tracks gives the opportunity to extend the robot's workspace, but on the other hand there is an additional error component in the robotic system which must not only be analysed but in the best case compensated.

The following document deals with a special task of analyzing of linear tracks for industrial robots. Dependent on an earlier article [15] there exists a method for a highly accurate integration of track profiles on the robot control. This is partly based on

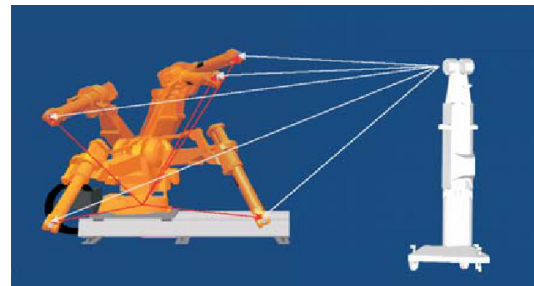


Fig. 1. Identifying of the robots base via 10 measured points

the interpolation of one, in single sampling positions measured, three-dimensional track profile to get a continous description of the linear track. For this the

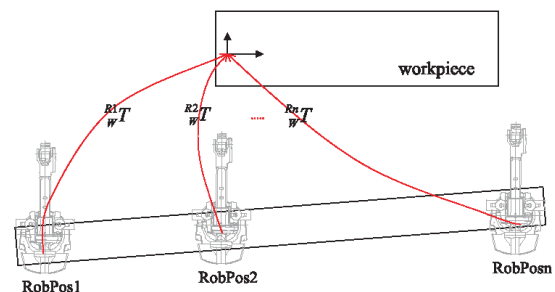


Fig. 2. Continous description of the linear track

robot's base frame is identified at several equidistant positions by measuring 10 different points with an external measurement system (Leica Laser Tracker

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LTD800), figure 1 illustrates this situation. After identifying the single track coordinate systems an interpolation between the sampling points creates a continuous description of the track profile (figure 2).

Based on this interpolation further steps for the calculation of the correction values are done so that the error of the interpolation has an effect on the accuracy of the whole integration method.

The identification of the best interpolation method for getting the continuous description of the track file is the basic topic of this document. Based on this, an experimental attempt was selected in which first a simulated interpolation on some test data was done using MATLAB. The next step was the creation of a test built up for measuring the real track profile. After getting the real measurement data, three different interpolation methods were tested and the dependent deviations calculated.

## II. TESTED INTERPOLATION METHODS

For the interpolation of the 3-dimensional movement, each direction was separately interpolated. Three different interpolation methods for the interpolation between the sampling points in one direction of the linear track were tested. The basic idea of all three methods was to find a continuous function  $g(x)$  which approximates a given tabulated function  $f(x)$  so that

$$(f(x) - g(x))^2 < \varepsilon, \quad \varepsilon > 0 \quad (1)$$

The interpolation function  $g(x)$  is equal to the tabulated function  $f(x)$  in the  $N$  given tabulated points  $x_i$ :

$$f(x_i) = g(x_i), \quad i = 0, 1, \dots, N-1 \quad (2)$$

### A. Polynomial interpolation

Basing on the Weierstrass Theorem the task of the polynomial interpolation is to find a polynomial  $p_n(x)$  of degree  $N$  which agrees with a function  $f(x)$  given through  $N$  tabulated values, so

$$p_n(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0 \quad (3)$$

There are different methods for calculating this  $p_n(x)$ , such as Lagrange, Newton, Bessel, Stirling or many others. These methods are well-known in the literature and for the purposes of calculating one polynomial interpolation for  $p_n(x)$  the least squares method was used in this paper.

### B. Trigonometric interpolation

The second tested interpolation method is the trigonometric interpolation. Based on the Fast Fourier Transformation the interpolating function  $t_n(x)$  which interpolates the function  $f(x)$  given in tabulated and

equidistant sampling points  $(x_i, f_i)$  consist of an addition of trigonometric components:

$$t_n(x) = a_0 + \sum_{j=1}^n a_j \cos(jx) + \sum_{j=1}^n b_j \sin(jx) \quad (4)$$

The coefficients of the trigonometric function  $t_n(x)$  are calculated by the FFT-algorithm.

### C. Cubic spline interpolation

Using this interpolation method, not one function with a large polynomial order, but several functions with degree of 3 are used to interpolate the given tabulated function  $f(x)$ . Between every two sampling points a different smooth and also smooth in the first derivate spline-function is defined:

$$S_i(x) = a_i + b_i(x - x_i) + c_i(x - x_i)^2 + d_i(x - x_i)^3 \quad (5)$$

The concatenation of the spline-functions gives the interpolating function.

## III. INTERPOLATION TEST ON NON-REAL MEASUREMENT DATA

The first step of the interpolation analysis was a simulation on artificial measurement data with MATLAB. For this a special MATLAB batch file was programmed, which created different interpolations on a given set of values for the measurement function  $f(x)$ . The 3-dimensional movement of the robot's TCP was interpolated separately for each axis x, y and z. After the successful simulation the real measurement data can be given into the system.

As can be seen, in the programmed MATLAB file the three different interpolation methods polynomial, trigonometric and cubic spline interpolation are used to create prediction how the different methods behave during interpolation.

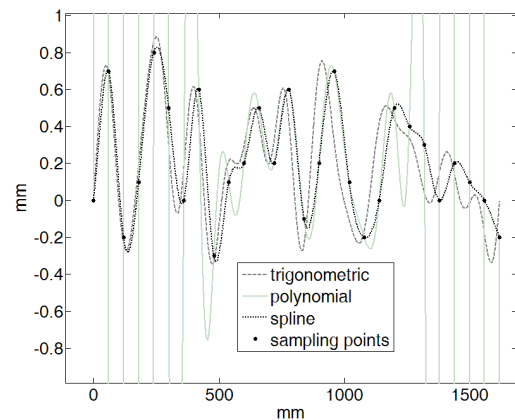


Fig. 4. Simulated interpolation

Figure 4 shows the results of the three interpolations with test values of 28 sampling positions. The differences between the single values caused by a simulated inaccuracy are 1 mm.

#### IV. MEASUREMENT OF THE REAL TRACK MOTION

##### A. Measurement device

The used measurement system for the needed measurement tasks like scanning the track motion, determining the particular track coordinate systems or identifying of the robots accuracy is represented by a Leica Laser Tracker LTD800 (figure 5). With this measurement device it is possible to do touchless measurements of 3-dimensional points in a range up to 80 meters. The measurement uncertainty of a coordinate is given by  $10 \mu\text{m} + 5 \mu\text{m/m}$  (the accuracy is dependent on the measurement distance) with a possible maximum measurement rate of 3000 points per second.



Fig. 5. Leica laser tracker LTD800

Using the world's most accurate absolute distance meter for getting the distance between laser track and measuring point ( $25 \mu\text{m}$  within 40m) and two built-in precision encoders for horizontal and vertical angle measurements, it is a highly accurate measurement system and common in measurement tasks for aircraft and automobile industries.

##### B. Robot accuracy

For proving the measurements for identification of the linear track profile, the accuracy of the robot was tested dependent on an european standard (DIN EN ISO 9283). In this case interesting values were the positional accuracy ( $AP_p$ ) and the repeating accuracy ( $RP_1$ ), to show that the robot can be used as a rigid device when moving with its repeating accuracy. As described in the standard, the positional accuracy is calculated by:

$$AP_p = \sqrt{(\bar{x} - x_c)^2 + (\bar{y} - y_c)^2 + (\bar{z} - z_c)^2} \quad (6)$$

where  $\bar{x}, \bar{y}, \bar{z}$  are the mean values of the coordinates form 30 different movements to one programmed point  $p_c(x_c, y_c, z_c)$ . The repeating accuracy then is:

$$RP_1 = \bar{l} + 3S_l \quad (7)$$

$$\bar{l} = \frac{1}{n} \sum_{j=1}^n \sqrt{(x_j - \bar{x})^2 + (y_j - \bar{y})^2 + (z_j - \bar{z})^2} \quad (8)$$

$$S_l = \sqrt{\frac{\sum_{j=1}^n (l_j - \bar{l})^2}{n-1}} \quad (9)$$

where  $x_j, y_j, z_j$  are the coordinates of one measured point and  $S_l$  is the standard deviation. Figure 6 depicts the positional and repeating accuracy of the used robot.

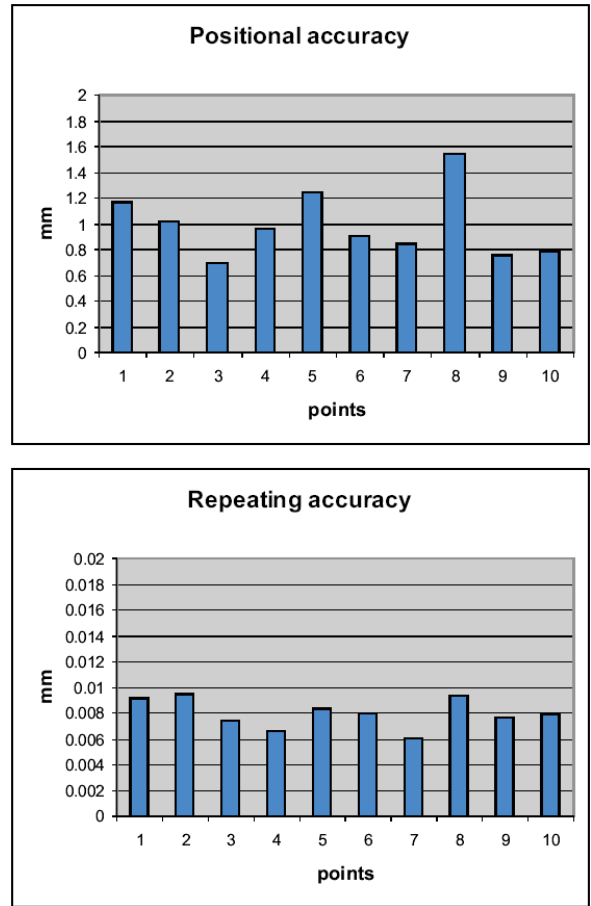


Fig. 6. Accuracy of the used robot

### C. Measurement system

For analysing the track profile the robot can be used as a rigid device because of its high repeating accuracy. To get the sampling points for the interpolation, the robot was measured at 28 different equidistant linear track positions in one programmed control point (figure 7).

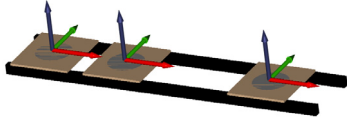


Fig. 7. Measurement at 28 sampling points

So the robot did not move but only the linear track caused potential positional errors. Figure 8 gives an overview about the whole measurement system setup. To compare the interpolating function with real

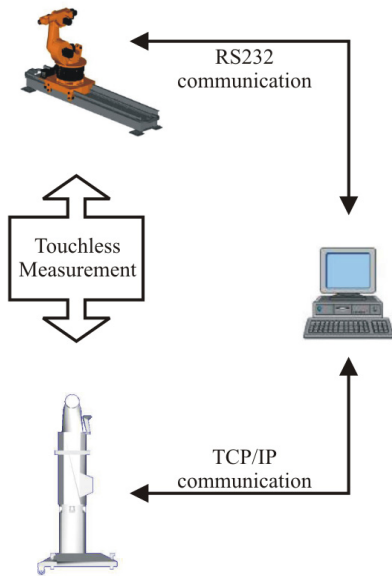


Fig. 8. Build up of the measurement system

measurement data, the robot was not only measured in the discrete sampling points, but also a quasi-continuous scan of the robots TCP during a linear track movement was done. It is quasi-continuous because the measurement rate was set to 1000 measurements per second, therefore the leftover discretisation error can be neglected.

### V. ERROR CALCULATION BETWEEN REAL MOVEMENT AND INTERPOLATION

Comparing the interpolation functions with the real measured values the corresponding error between them decides the quality of the used interpolation method. To get this error the difference  $d_{error}$  between every measurement value  $p_i$  of the quasi-continuous scan and the corresponding value of the interpolation function  $f_{ipo}(x_i)$  is calculated by:

$$d_{error,i} = \left| p_i(\xi) - f_{ipo,\xi}(x_i) \right| \quad (10)$$

where  $p_i(\xi)$  is the x,y or z-coordinate of one measurement value of the quasi-continuous scan. Figure 9 shows the robots TCP movement represented by the measured sampling points and the three different

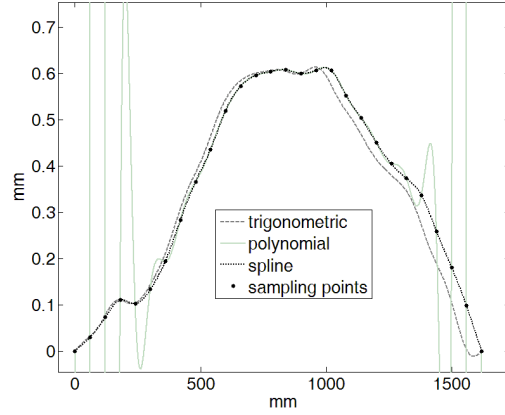


Fig. 9. Interpolation functions and robot movement scan

corresponding interpolation methods. The polynomial interpolation (solid line) confirms the results of section III. A swinging and non-exact interpolation is caused by a polynomial of a high order. In difference to this, the trigonometric interpolation (dashed line)

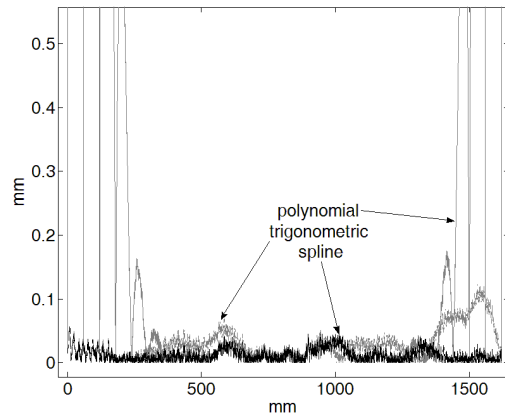


Fig. 10. Deviation between interpolation and real robot movement

and the cubic spline interpolation (dotted line) present a better approximation behaviour to the real TCP movement.

Figure 10 shows the differences  $d_{error,i}$  between interpolation function and a continuous measurement of the robot's TCP movement. Depending on the maximums of the error functions shown in figure 10, the cubic spline interpolation is the most accurate of the tested interpolation methods.

## VI. CONCLUSIONS

Regarding to the lowest difference between the interpolation function and real TCP movement, the cubic spline interpolation was chosen to create the continuous description of the track motion. Even the simple construction of the particular spline functions from one sampling point to another offers an efficient implementation in the robot control for further steps in the integration of track motions in the motion control of industrial robots.

The interpolation of the sampling points on the linear track is an important step in the achieving of highly accurate robotic systems. The quasi-continuous scan of the robot's TCP position during a movement along the linear track could awake the impression, that due to this exact measurement of the TCP, no interpolation is needed. But it gives only information for one robot joint configuration. To get a general prediction about arbitrary robot positions, e.g. for correcting robot programs the base frame positions of the robot have to be interpolated. For this, the calculated base frame coordinates, given through the measurement of the robot in 10 different points on one linear track position are taken as sampling points for the interpolation and not the measured points. For comparing the results for this interpolation test it was necessary to have one robot position and not the robot base interpolated.

A further step in this project will be the analysing of three-dimensional interpolation methods to avoid inaccuracies caused by the disjunction of the 3-dimensional movement to 3x1-dimensional movements.

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