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Direction-of-arrival algorithms: TAM and R-ESPRIT

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Abstract – since wireless communications began to take an abrupt rise, with the development of techniques which were introduced for the growth of data transfer, antenna systems were considered to be the ideal choice to obtain the wanted results. Nevertheless these techniques required the development of certain algorithms also, to facilitate the recognition of the signals at the reception side (direction of arrival (DOA), phase, frequency, etc.)

The object of this article two of these DOA algorithms, with the focus on the precision of the identification of signals at the reception end. The performance evaluation is based on the simulation of the algorithms and the estimation of the parameters. The antenna system, or antenna array, used for these simulations is a linear array with a variable number of elements.

Keywords: R-ESPRIT, TAM, DOA estimation

I. INTRODUCTION

Determining the direction-of-arrival (DOA) of any signal of interest has long been of great interest to the research community. It is often required to identify the DOA of signals emanating from every possible direction. This is important for military as well as for civilian applications. The directions to targets are usually expressed by the DOA of the transmitted signals from the sources. Several models based DOA estimation are given in the literature.

If the DOA of the desired signal is known, then adaptive beamforming algorithms can be performed to minimize the signal power of the interference, which will in turn maximize the power of the signal of interest. DOA estimation is one of the main functional requirements for direction-finding smart antennas in future generation mobile and stealth communication systems.

This article will focus on two DOA estimation methods: the TAM and R-ESPRIT algorithms.

II. THE R-ESPRIT ALGORITHM

The R-ESPRIT is considered as being one from the ESPRIT family. This new noniterative method based on a real-valued signal model and brings a spectacular reduction in the number of operations required to compute the frequency estimates. The algorithm was proposed and presented in detail in [1] and [2]. Here, however, we will consider here the description given in [3], presenting the main aspects of this algorithm.

<u>The signal model</u>

The input signal consists in a number r of sinusoidal signals embedded in white Gaussian noise:

$$x(t) = \sum_{k=1}^{r} s_k \sin(t\omega_k + \varphi_k) + n(t) \quad (1)$$

where s_k is the amplitude, ω_k is the angular frequency of the k^{th} sinusoid and $n(t)$ representations the corrupting additive zero-mean white noise. The phases $\{\varphi_k\}_{k=1}^{r}$ are random variables uniformly distributed in the $[-\pi, \pi]$ interval.

The compact subspace representation dedicated for real valued sinusoids differs from the classical complex-valued signal model [4]. We have to obtain an alternative snapshot vector so that its noise-free part lies in a subspace of dimension r. To that aim, we will introduce the following input vectors:

$$x_f(t) \triangleq [x(t) \dots x(t+n-1)]^T \quad (2)$$

$$x_b(t) \triangleq [x(t-1) \dots x(t-n)]^T \quad (3)$$

$$x_r(t) \triangleq \frac{1}{2}[x_f(t) + x_b(t)]^T \quad (4)$$

where the snapshot vector dimension $n>2r$.

From the above definitions we obtain:

$$x_r(t) = A_r s_r(t) + n_r(t) \quad (5)$$

Where $s_r(t)$ is an $r \times 1$ vector given by

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$$s_r(t) = \begin{bmatrix} a_1\cos\{\omega_1 t + \phi_1^*\} \\ \vdots \\ a_r\cos\{\omega_r t + \phi_r^*\} \end{bmatrix} \quad (6)$$

where

$$\phi_k^* = \phi_k \quad \frac{1}{2}\omega_k \text{ for } 1 \le k \le r \quad (7)$$

A_r is an $n \times r$ matrix given by

$$A_r = \begin{bmatrix} \cos\left(\frac{\omega_1}{2}\right) & \cdots & \cos\left(\frac{\omega_r}{2}\right) \\ \cos\left(\frac{3\omega_1}{2}\right) & \cdots & \cos\left(\frac{3\omega_r}{2}\right) \\ \vdots & \ddots & \vdots \\ \cos\left\{\left(n-\frac{1}{2}\right)\omega_1\right\} & \cdots & \cos\left\{\left(n-\frac{1}{2}\right)\omega_r\right\} \end{bmatrix} \quad (8)$$

The noise snapshot vector $n_r(t)$ in this modified model is given by

$$n_r(t) = \frac{1}{2}\{n_c(t) + n_b(t)\} \quad (9)$$

where

$$n_c(t) \triangleq [n(t) \ldots n(t+n-1)]^T \quad (10)$$

$$n_b(t) \triangleq [n(t-1) \ldots n(t-n)]^T \quad (11)$$

One can show that A_r is a full column rank matrix. The important fact here is that the noise-free part of $x_r(t)$ lies in an r-dimensional subspace that is different from the complex-valued data model, where the dimension of the signal subspace is 2r. Further on, let us introduce

$$P_r \triangleq E\{s_r(t)s_r^T(t)\} \quad (12)$$

The noise vectors $n_c(t)$ and $n_b(t)$ are random vectors, mutually independent, with

$$E\{n_r(t)n_r^T(t)\} = \left(\frac{\sigma^2}{2}\right) I_n \quad (13)$$

where σ^2 is the noise variance.We obviously have that:

$$R_r \triangleq E\{x_r(t)x_r^T(t)\} = A_r P_r A_r^T + \left(\frac{\sigma^2}{2}\right) I_n$$

(14)

We may then consider the eigenvalue decomposition

$$R_r = S_r \Lambda_r S_r^T + G_r \Sigma_r G_r^T \quad (15)$$

where Λ_r is an $r \times r$ diagonal matrix containing the r dominant eigenvalues of R_r on the diagonal. The $n \times r$ matrix S_r is composed of the corresponding left eigenvectors.in the same perspective; Σ_r is a diagonal matrix containing the remaining $n - r$ eigenvalues of R_r. The $n \times (n - r)$ matrix G_r is composed of the corresponding left eigenvectors. The columns of G_r are orthogonal to those of S_r.

One can show that

$$S_r = A_r C_r \quad (16)$$

where

$$C_r = P A_r^T S_r \left\{\Lambda_r - \frac{\sigma^2}{2} I_n\right\}^{-1} \quad (17)$$

The columns of S_r form an orthonormal basis of the column space of A_r.

The basic idea is to make use of two $(n-2) \times n$ Toeplitz matrices:

$$T_r^{(1)} = \begin{bmatrix} 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & 0 & 1 & 0 \end{bmatrix} \quad (18)'$$

$$T_r^{(2)} = \frac{1}{2}\begin{bmatrix} 1 & 0 & 1 & 0 & \cdots & 0 \\ 0 & 1 & 0 & 1 & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & 1 & 0 & 1 \end{bmatrix} \quad (18)''$$

From the definition of A_r matrix we can write:

$$T_r^{(2)} A_r = T_r^{(1)} A_r D_r \quad (19)$$

where D_r is the following diagonal matrix:

$$D_r = \mathrm{diag}\{\cos(\omega_1), \ldots, \cos(\omega_n)\} \quad (20)$$

Let us also introduce the following matrix:

$$\Phi_r = C_r^{-1} D_r C_r \quad (21)$$

Then, the algorithm may be derived as follows:

1. It is first required to estimate $\hat{S}_r$ from the input data
2. This estimate will be used in estimating $\hat{\Phi}_r$

$$\hat{\Phi}_r = (T_r^{(1)}\hat{S}_r)^{-1} T_r^{(2)} \hat{S}_r \quad (22)$$

3. The eigendecomposition of $\hat{\Phi}_r$, will lead us to $\hat{D}_r$
4. Knowing $\hat{D}_r$, frequency values easily result from:

$$\omega_k = \cos^{-1}\{\hat{D}_r(k,k)\} \qquad k=1,\ldots,r \quad (23)$$

This finally gives the frequency estimates. The dimension of the signal subspace is reduced to r from $2r$ i.e. the case of traditional ESPRIT method for r real-valued sine waves.

Simulations

Running repeated simulations to estimate the direction of arrival of two signals, the results were noticeably satisfactory (Figure 1).

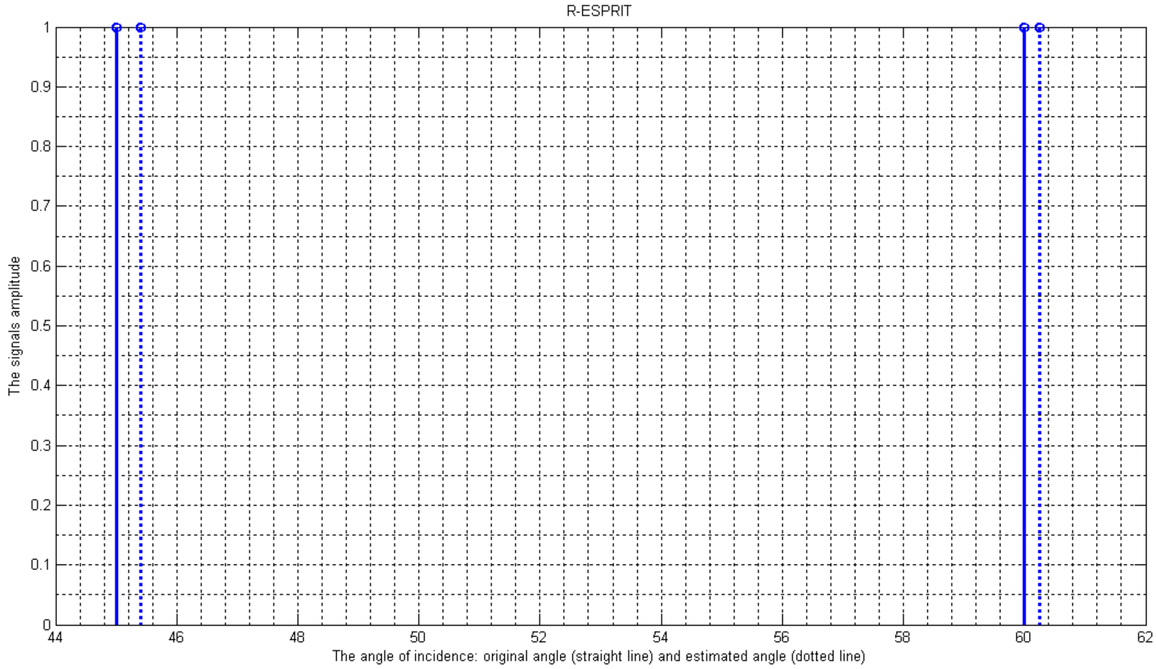

Figure 1. The R-ESPRIT alghoritm. The straight line samples reprezent the real values, while the dotted line samples are the estimated values (punctual example)

The simulations were done, considering the distance between the antenna elements of λ/2, 8 elements, the signals amplitude is 1, with the value of the ***SNR=0***. Simulations were done, estimating two signal sat arrival.

The results obtained after running the simulations present a precision of 0.37%, meaning the estimated value is off by +/- 0.37% from the real value.

If we run several simulations, trying to estimate only on angle (in this case), the accuracy is more visible in following diagram:

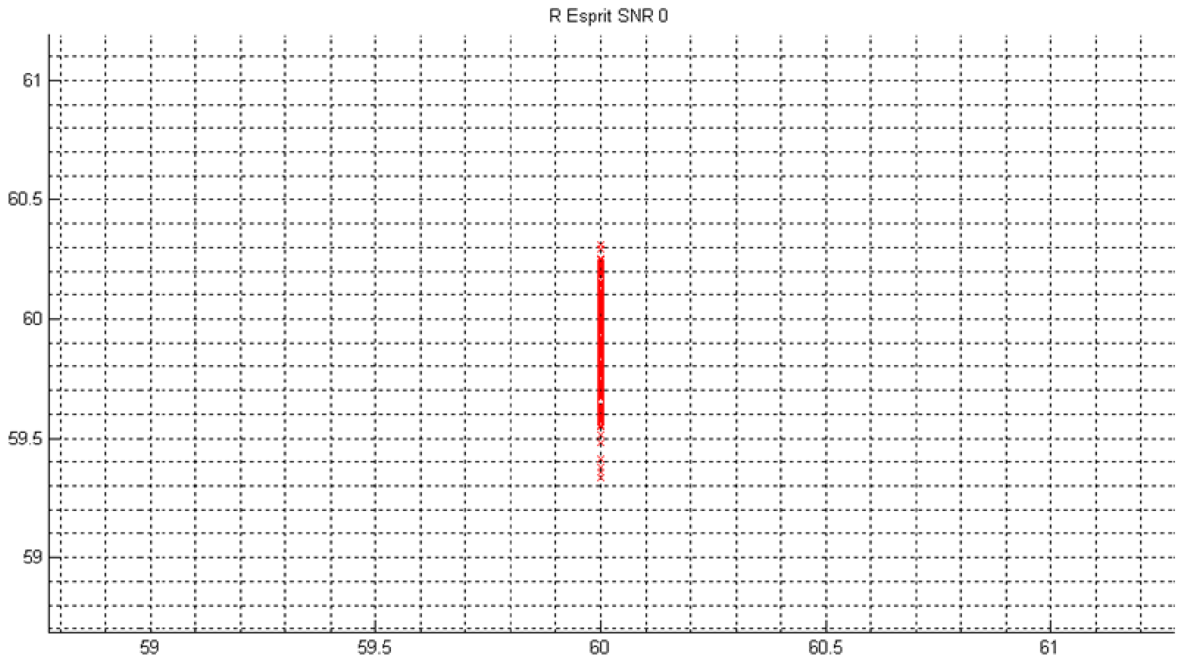

Figure 2. The R-ESPRIT alghoritm. Angle to estimate 60˚, SNR=0. The ***y***-axis represents the estimation values.

In case we modify the SNR (SNR=-10), while running several simulations, again trying to estimate on angle of arrival, the results are visible in the following diagram:

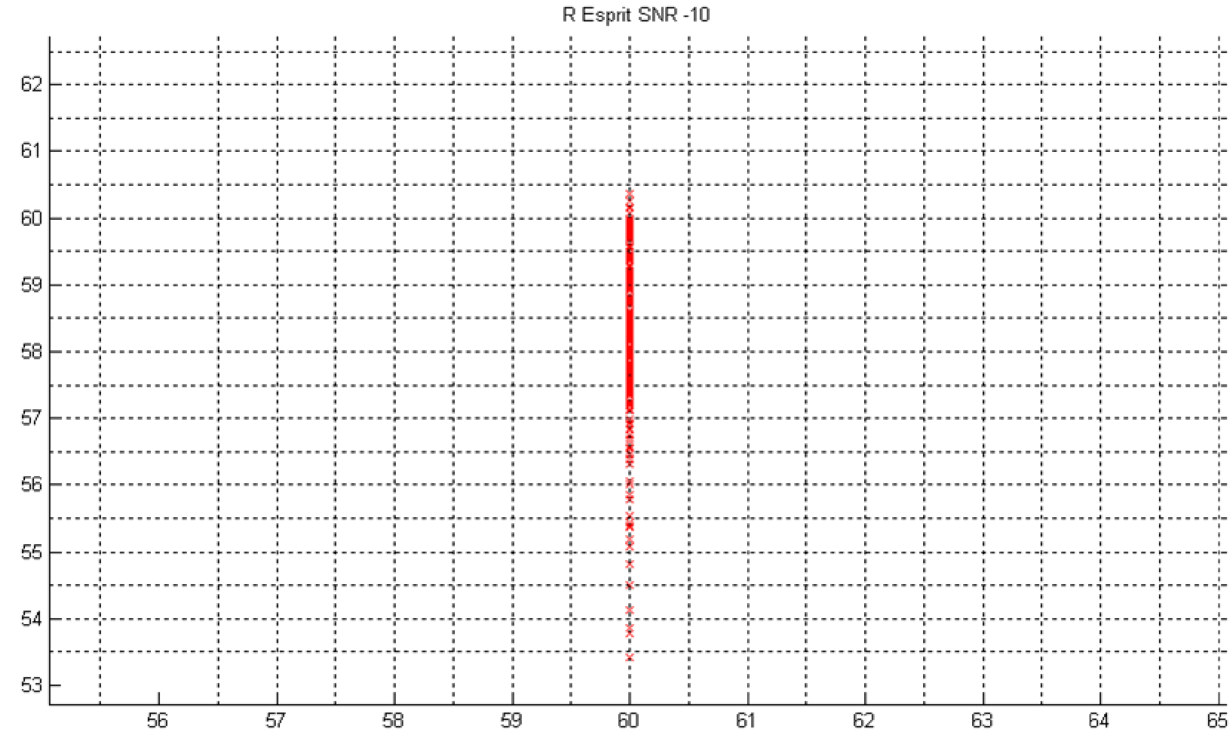

Figure 3. The R-ESPRIT alghoritm. Angle to estimate 60˚, SNR=-10. The *y*-axis represents the estimation values.

Most of the estimated values are grouped around the real value: 60˚; this way the error with which the estimations are done has a reasonable value, considering the SNR. Comparing this error to the estimations done at ***SNR=0***, we can see that there is a difference in the precision: +/- 1,50.

III. THE TAM ALGORITHM

This method is originally a method of harmonic analysis. However, it can be easily transposed to DOA estimation problem, under the condition that the antennae be identical, omnidirectional and equispaced. We are going to present the method in this latter case [9].

Let us assume m non-totally coherent plane waves are impinging on a linear array of N sensors. For the k-th snapshot, the observation vector is given by:

$$z_k^T = \left[\sum_{i=1}^{m} p_i^{(k)}, \sum_{i=1}^{m} p_i^{(k)} e^{j\omega_i}, \ldots, \sum_{i=1}^{m} p_i^{(k)} e^{j(N-1)\omega_i}\right] + N_k^T$$
(24)

where for plane waves of frequency *f*, $\omega_i = 2\pi d f \sin(\theta_i)/c$, with c the velocity of propagation, d the element spacing, and θ_i the angle of incidence of the i-th wave. N_k is assumed to be a complex, zero-mean, circular, Gaussian vector with independent elements of variance $\sigma^2 p_i^{(k)}$ are the complex random amplitudes of the plane waves.

If we note the parenthesis […] with z_k^T, we can write

$$Z_k^T = z_k^T + N_k^T \quad (25)$$

Let ***RZZ*** be the observation covariance matrix:

$$RZZ = E[Z_k Z_k^+] = E[z_k z_k^+] + \sigma^2 I_N = Rzz + \sigma^2 I_N$$
(26)

Let S_N the ***N x m*** Van der Monde matrix associated with the sensors:

$$S_N = \begin{bmatrix} 1 & \cdots & 1 \\ e^{j\omega_1} & \cdots & e^{j\omega_m} \\ \vdots & \ddots & \vdots \\ e^{j(N-1)\omega_1} & \cdots & e^{j(N-1)\omega_m} \end{bmatrix} \quad (27)$$

We have $z_k = S_N a_k$, with $a_k = [p_1^{(k)}, \ldots, p_m^{(k)}]^T$. Therefore $Rzz = S_N A S_N^+$, where ***A*** is the amplitude covariance matrix. A is full-rank, since the sources were supposed to be non-totally coherent. Thus the rank (***Rzz)=m*** and range(***Rzz***)= (S_N). Let $U\Sigma U^+$ be a singular value decomposition (SVD) of ***Rzz***, where ***U*** and ***Σ*** are ***N x N*** matrices. Since ***Rzz*** is rank-m, we

can write $R_{ZZ} = U_S \Sigma_S V_S^H$. Σ_S is Σ truncated to a m x m dimension.
Continuing this logic we can draw the conclusion that S_N and U_S are nearly equal:

$$S_N = U_S P \quad (28)$$

Therefore we will obtain the generic equation:

$$\begin{bmatrix} 1 & \cdots & 1 \\ e^{j\omega_1} & \cdots & e^{j\omega_m} \\ \vdots & \ddots & \vdots \\ e^{j(N-2)\omega_1} & \cdots & e^{j(N-2)\omega_m} \end{bmatrix} \begin{bmatrix} e^{j\omega_1} & 0 & 0 \\ 0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & e^{j\omega_m} \end{bmatrix} = \begin{bmatrix} e^{j\omega_1} & \cdots & e^{j\omega_m} \\ e^{j2\omega_1} & \cdots & e^{j2\omega_m} \\ \vdots & \ddots & \vdots \\ e^{j(N-1)\omega_1} & \cdots & e^{j(N-1)\omega_m} \end{bmatrix} \quad (29)$$

We execute the following annotations:

$$S\downarrow = \begin{bmatrix} 1 & \cdots & 1 \\ e^{j\omega_1} & \cdots & e^{j\omega_m} \\ \vdots & \ddots & \vdots \\ e^{j(N-2)\omega_1} & \cdots & e^{j(N-2)\omega_m} \end{bmatrix} \quad (30)'$$

$$D = \begin{bmatrix} e^{j\omega_1} & 0 & 0 \\ 0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & e^{j\omega_m} \end{bmatrix} \quad (30)''$$

$$S\uparrow = \begin{bmatrix} e^{j\omega_1} & \cdots & e^{j\omega_m} \\ e^{j2\omega_1} & \cdots & e^{j2\omega_m} \\ \vdots & \ddots & \vdots \\ e^{j(N-1)\omega_1} & \cdots & e^{j(N-1)\omega_m} \end{bmatrix} \quad (30)'''$$

$S\downarrow$ and $S\uparrow$ can be obtained from S_N , by deleting the last and first row. Therefore, we can write:

$$S\downarrow = U_S\downarrow P \quad (31)'$$
$$S\uparrow = U_S\uparrow P \quad (31)''$$

We can also write:

$$U_S\downarrow (PDP^{-1}) = U_S\uparrow \quad (32)$$

The eigenvalues of PDP^{-1} are of course the $e^{j\omega_1}$. Thus letting $F = PDP^{-1}$ completes the proof in the case of a noiseless experiment or in an asymptotic case where ***RZZ***; we will tale for F the LMS solution of the equation:

$$U_S\downarrow F = U_S\uparrow \quad (33)$$

The eigenvalues will be approximations of $\{e^{j\omega_1}, \ldots, e^{j\omega_m}\}$.

<u>Simulations</u>

<u>Running repeated simulations</u> to estimate the direction of arrival of two signals, the results were noticeably satisfactory (Figure 4).

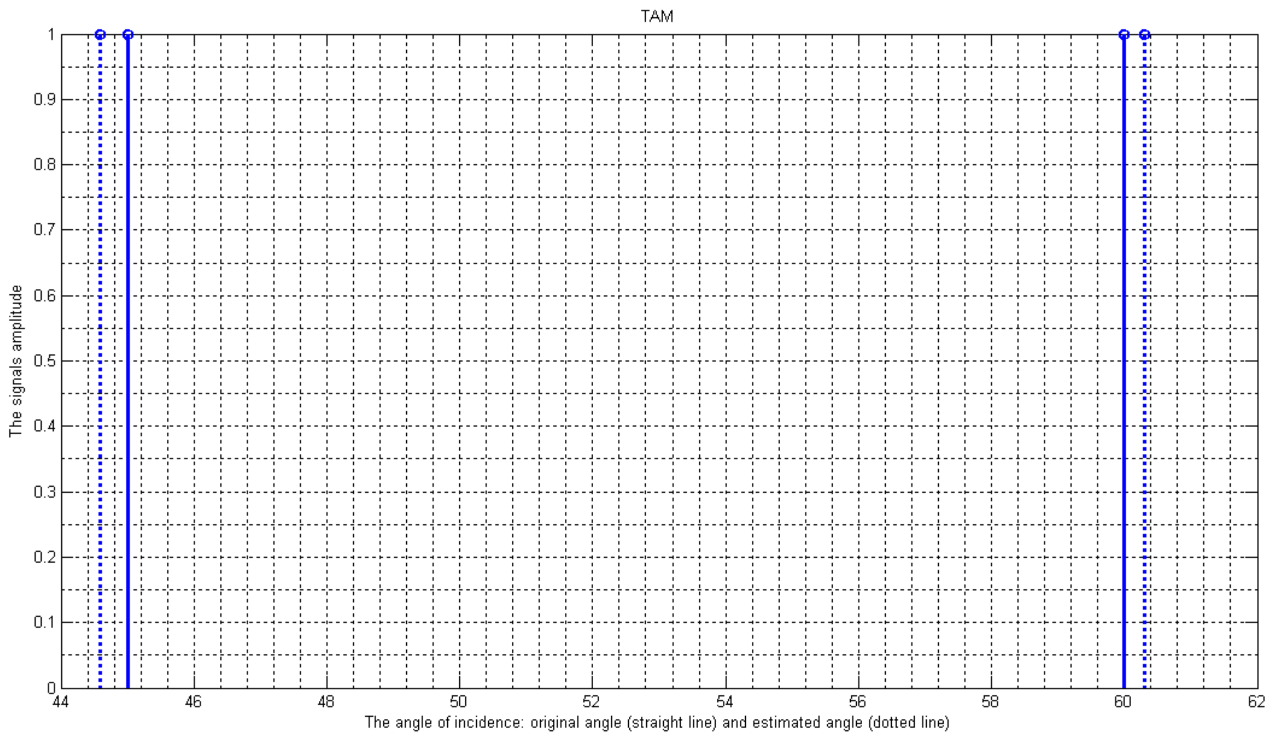

Figure 4. The TAM alghoritm. The straight line samples reprezent the real values, while the dotted line samples are the estimated values (punctual example)

The simulations were done, considering the distance between the antenna elements of λ/2, 8 elements, the signals amplitude is 1, with the value of the ***SNR=0***. Simulations were done, estimating two signals at arrival.

The results obtained with TAM are similar with the ones obtained with R-ESPRIT, with a small difference: the results obtained after running the simulations present a precision of 0.39%, meaning the estimated value is off by +/- 0.39% from the real value.

If we run several simulations, trying to estimate only on angle (in this case), the accuracy is more visible in following diagram:

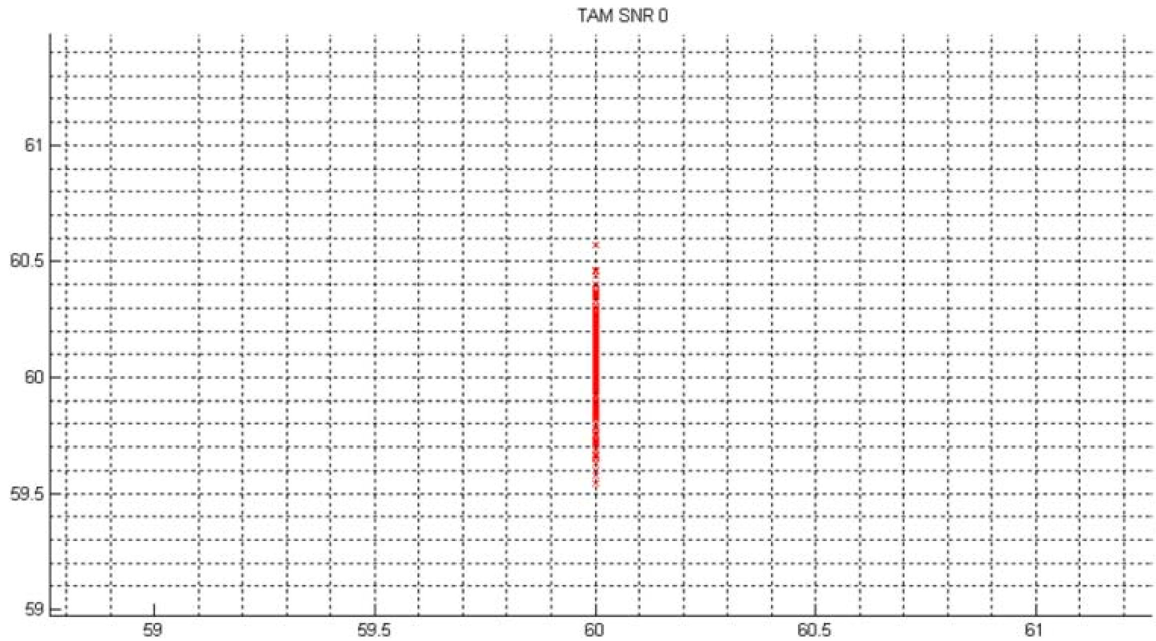

Figure 5. The TAM alghoritm. Angle to estimate 60˚, SNR=0. The ***y***-axis represents the estimation values.

In case we modify the SNR (SNR=-10), while running several simulations, again trying to estimate on angle of arrival, the results are visible in the following diagram:

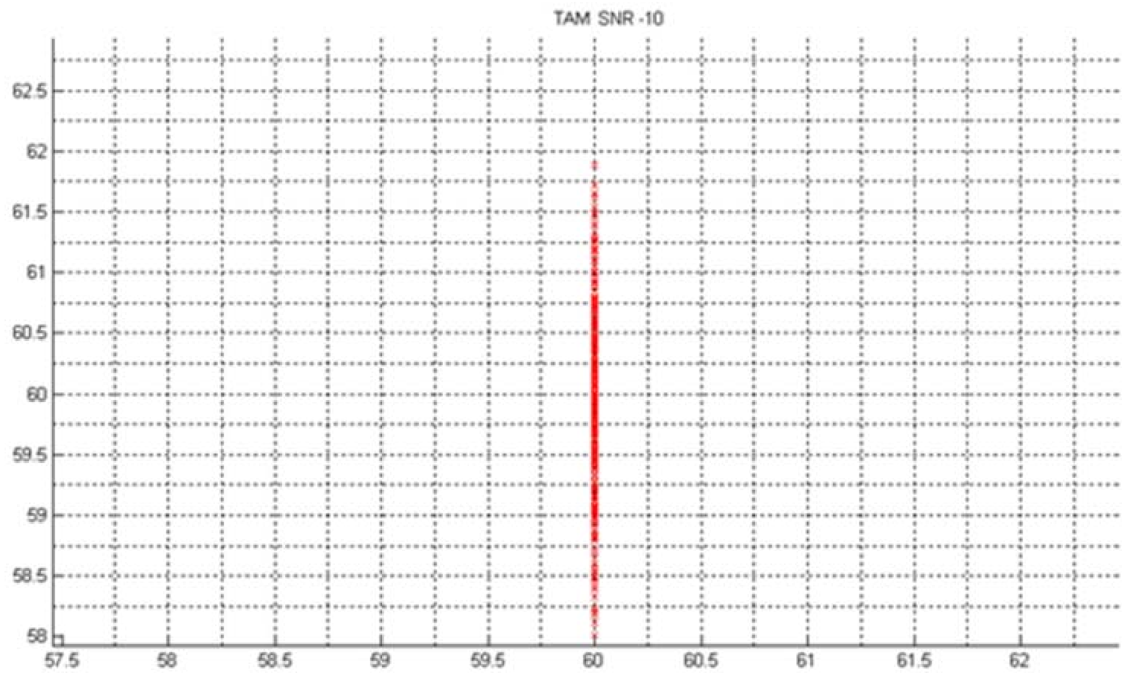

Figure 6. The TAM alghoritm. Angle to estimate 60˚, SNR=-10. The ***y***-axis represents the estimation values.

Most of the estimated values are grouped around the real value: 60˚; this way the error with which the estimations are done has a reasonable value, considering the SNR. Comparing this error to the estimations done at ***SNR=0***, we can see that there is a difference in the precision: +/- 1,73.

IV. CONCLUSIONS

From all of the above we can see that for the different type of algorithms developed in the previous years, there are positive and negative sides. Nevertheless the opportunities offered by them are highly valuable and the applicability is vast.

Comparing the two presented algorithms, the R-ESPRIT presents better estimation values than the TAM algorithm. Nevertheless both TAM and R-ESPRIT are two of the best variants for estimating the direction-of-arrival.

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