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Broadband measurement of the refractive index using microstrip lines

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Abstract – Measurement methods based on Vector Network Analyzer, for the determination of the refractive index of a medium, are described. The refractive index is obtained from the phase velocity of the radiofrequency signal along a microstrip line embedded in the considered medium. Simple structures are used for this purpose and advantages and disadvantages of different methods are discussed.

Keywords: microstrip lines, effective dielectric constant, Network Analyzer, education, microwaves

I. INTRODUCTION

Teaching electromagnetic course is usually an arduous task because of the new concepts to be transferred to the students. Description of the phenomena requires a good mathematical background and also, in the case when students understand the theoretical formulation, some of the concepts are not fully clarified from a practical point of view.

In such a context, this work has the aim to present some simple methods, well adapted as laboratory exercise for graduate courses, for the determination of the refractive index (RI) of a medium by means of a vector network analyzer (VNA).

The knowledge of the value of the RI n for a medium is important in different applications such as design of matching networks, phase shifters, etc.

For this purpose, we will consider a microstrip line embedded in the medium for which we want to evaluate the RI, and measurements on the phase of the reflection/transmission coefficients will allow us to determine the effective value of the dielectric constant ε_{eff}. The RI is related to this latter by:

$$n = \sqrt{\varepsilon_{eff}}$$

since it is defined as the ratio of the velocity of light in free space and the phase velocity in the considered medium.

Although the characteristic impedance of a microstrip line depends on ε_{eff} and on the ratio between the width of the line and heigth of the dielectric substrate

$$Z_{\infty} = Z_{\infty}(\varepsilon_{eff}, w/h)$$

in a relatively large frequency range, the effect of frequency on the characteristic impedance can be neglected.

Because of this dependence, in the following we will consider the general case when the microstrip line impedance does not match to the reference impedance of the VNA port. The effect of this mismatching is a non linear variation of the phase of the reflection coefficient versus frequency. From the maximum deviation with respect to a linear variation, one can compute the value of the line impedance as well.

II. DESCRIPTION OF THE MEASUREMENT METHODS

In this section we will describe various methods for the determination of the RI of a medium with the microstrip line. The described methods can be applied for single layer structures as well as multilayer ones. In this latter case, the value of the RI is different from layer to layer.

Since this work is essentially an educational one, we will start with the description of a simple but less accurate method based on an absolute measurement. It will be followed by different schemes, where using relative measurements some of the effects that contribute to the inaccuracy can be eliminated.

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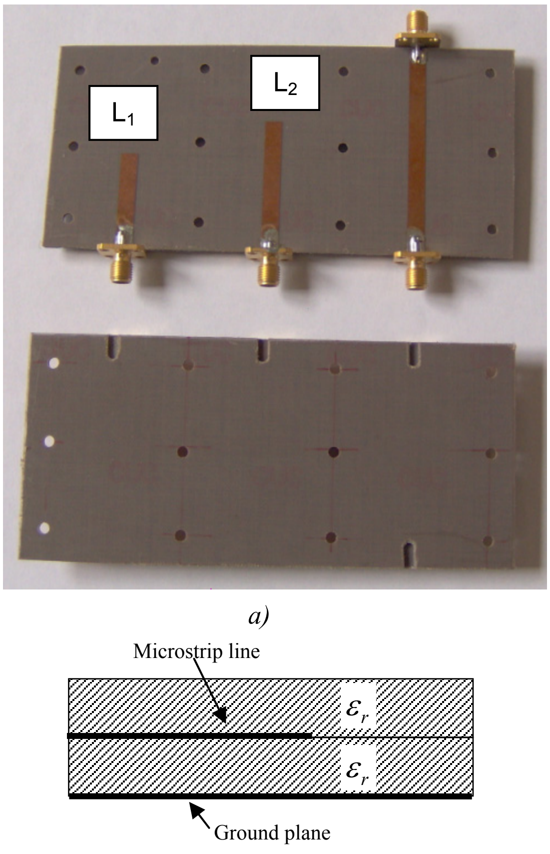
We note, that for the case of a single dielectric layer, an approximate expression for the effective value of the dielectric constant is [2]:

$$\varepsilon_{eff} = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2}\left(1 + 10\frac{h}{w}\right)^{-0.5}$$

Measurements results can be compared with this value.

The structure used for all the reported measurements is presented in Fig.1. The microstrip line lies between two Arlon Cuclad^{TM} dielectric sheets with ε_r=2.3, both of a height h=1/16" (1.6 mm). The lengths of the two open end lines (measured from the edge of the dielectric board) are L_1=26 mm and L_2=36 mm respectively.

In the figure also a thru is present. A second one, of different length, was built separately.



a)

b)

Figure 1 Photograph (a) and sketch b) of the considered structure

In Fig. 2 a photograph of the microstrip line in the experimental setup is presented. For all cases, it is supposed that the VNA has been calibrated before starting the measurements. In particular, we considered 1601 points in the 200 MHz-5 GHz frequency range.

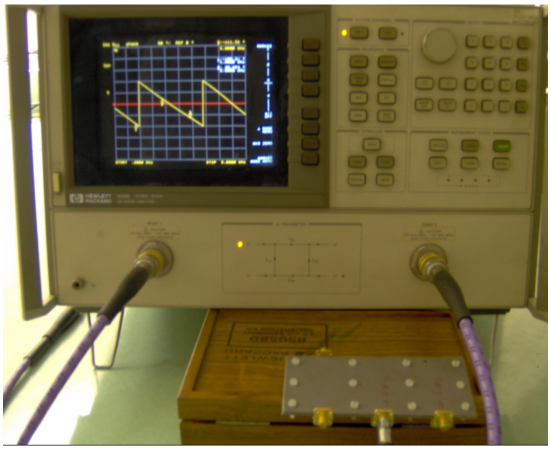


Figure 2 Photograph of the measurement setup

A. *Direct method*

The simplest method consists in measuring the phase of the reflection coefficient of an open-ended microstrip line, and introducing a shift of the reference plane, which is a feature of the VNA.

The considered circuit is depicted in Fig.3. The open ended microstrip line, of length ℓ, is fed by a coaxial-microstrip transition. The reference plane, defined during the calibration is somewhere inside the connector/transition.

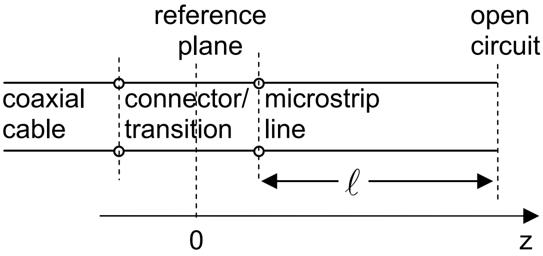


Figure 3 Sketch of the circuit for the direct method

An ideal open circuit has the phase of the reflection coefficients equal to 0°, but it is difficult to build due to radiation of the truncation and the fringing capacity. Moreover, we are interested in the fact that the *phase* at the terminal end is *constant* in a relatively wide frequency range.

Using the electrical delay option of the VNA, it is possible to shift the reference plane according to:

$$\Gamma(\ell_0) = \Gamma(0)e^{\pm 2jk_0\ell_0}$$

where k_0 is the *free space* propagation constant and ℓ_0 is the distance between the reference plane and the new section where the reflection coefficient is evaluated.

The + sign corresponds to a shift from the reference plane toward the load, while the – sign allows to move in the opposite direction.

Since the open circuit we are interested in is away from the reference plane, we will consider a *positive* phase shift.

By introducing the phase shift, we expect that, for a certain value of this electrical delay, the resulting phase to be constant in a certain frequency range. It is convenient to use a phase or Smith chart format for representing the measurements.

The resulting reflection coefficient is:

$$\begin{aligned}\Gamma(\ell_0, f) &= \Gamma(0, f)e^{+2jk_0(f)\ell_0} \\ &= \left|\Gamma(0, f)\right| e^{+2jk_0(f)\ell_0 + j\Phi(0,f)}\end{aligned}$$

where $\Phi(0, f)$ represents the phase of the measured reflection coefficient at the reference plane (z=0). The value of the electric delay that corresponds to this refers to a delay in free space. Denoting by ℓ_0 the value for which

$$2k_0(f)\ell_0 + \Phi(0, f) = const.$$

and by taking into account the definition of the RI, we can compute it as the ratio of this distance and the length ℓ of the line.

$$n = \frac{\ell_0}{\ell}$$

The main disadvantage of this method is that it requires the knowledge of the line length. Actually, ℓ includes the length of the line, the effect of the fringing field, and the distance inside the connector/transition from the reference plane to the beginning of the line. This latter distance is not known. To reduce the inaccuracy of the method, it is convenient to consider a relatively long line.

In Fig. 4, the phase of the reflection coefficient is reported for the line L$_2$. On the top, the electrical delay is equal to zero and a linear phase variation can be noticed. In the lower photograph, the uniform phase is obtained for an electrical delay equal to 438.12 ps. It corresponds to ℓ_0=131.34 mm. It is clear from the Smith chart, that it corresponds to an open circuit.

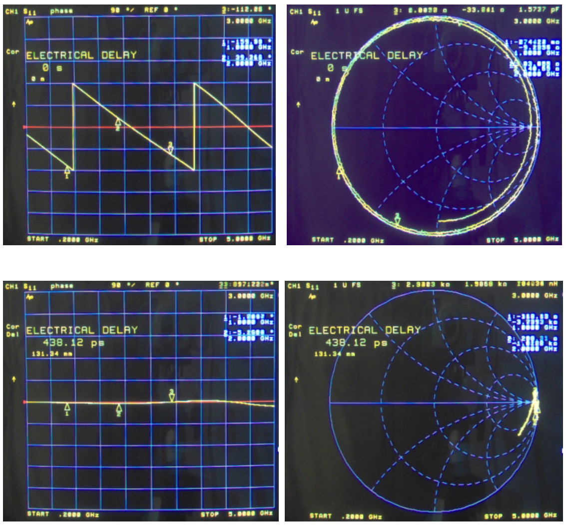


Figure 4 Zero electrical delay (top) and constant phase (bottom)

B. *Method based on the variation of the frequency*

Another fast method is based on the measurement of the phase of the reflection coefficient at two different frequencies. Denoting by 1 the reference frequency and by 2 a second value of the frequency, we have:

$$\begin{aligned}\Gamma_1 &= \Gamma(0)e^{-2jk_1\ell} = \Gamma(0)e^{-j\Phi_1} \\ \Gamma_2 &= \Gamma(0)e^{-2jk_2\ell} = \Gamma(0)e^{-j\Phi_2}\end{aligned} \tag{1}$$

where Γ_1 and Γ_2 represent the reflection coefficients at the input of the line for the two frequencies f_1 and f_2. The phase difference is:

$$\begin{aligned}-\Delta\Phi &= -\Phi_1 + \Phi_2 = 2k_2\ell - 2k_1\ell \\ &= 2\ell(k_2 - k_1) = \frac{4\pi}{c} n\ell\Delta f\end{aligned} \tag{2}$$

In the equation above we considered a lossless line $\left(k = \frac{2\pi}{\lambda}\right)$, and made use of the relations $n = \frac{c}{v_f}$ and $\lambda = \frac{v_f}{f}$ From eq. 2 it is therefore possible to obtain:

$$n = -\frac{c}{\ell}\frac{\Delta\Phi}{4\pi}\frac{1}{\Delta f}$$

It has to be noted, that in eq. (2), the phase difference is the total (unwrapped) one, and not only the value reported on the VNA: one has to consider a 2π additional term at each "jump", due to the representation within the interval $[-\pi, \pi]$.

As in the previous method, in this case we have an inconvenient due to the length of the line too. Actually, the equation makes use of the distance between the open end and the reference plane defined during the calibration. Since the line has to be connected by a connector, the length of the connector and the transition is not known in a precise way. This length can be quantified, or its effect can be eliminated by employing a relative measurement, as it will be described in the following paragraph.

C. *Method based on the variation of the length of the microstrip line*

Instead of varying the frequency, as in the previous paragraph, we can imagine a scheme with two microstrip lines of different lengths (see Fig. 5). In this case, the phase difference will originate from the difference between the two lines lengths at the same frequency:

$$n = -\frac{c}{\Delta\ell}\frac{\Delta\Phi}{4\pi}\frac{1}{f} \qquad (3)$$

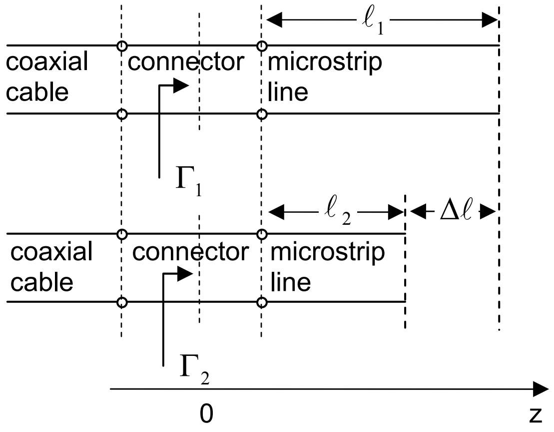


Figure 5 Sketch of the circuit for the method II. C

In Fig. 6, the behavior of two traces corresponding to two different line lengths (L1 and L2 in Fig. 1) for different values of the electrical delays are presented. The first photograph (top) refers to the absence of the electrical delay. The active trace, with the markers on it, corresponds to the shorter line. The second trace represents the phase of the longer line. It is clear from the photo that the longer line presents a higher variation of the phase for the same frequency range.

Introducing an electric delay and increasing its value up to ED_1 =349.88 ps (center photograph), the phase corresponding to line L1 presents azero mean value. The amplitude of the oscillations correspond to the mismatching between the 50 Ω reference impedance and the line impedance. By reporting this value of the reflection coefficient on a Smith chart, and by considering a circle with center in the center of the Smith chart and tangent to this line, the modulus of the reflection coefficient can be obtained. Since line impedance and reference impedance are real, the intersection of this circle with the real axis will represent the ratio between the two impedances.

Further increasing the electric delay, the phase corresponding to the L1 line will move from the zero mean value. We will find the same behavior for the L2 line for an electric delay of ED_2 =446.45 ps (bottom photograph).

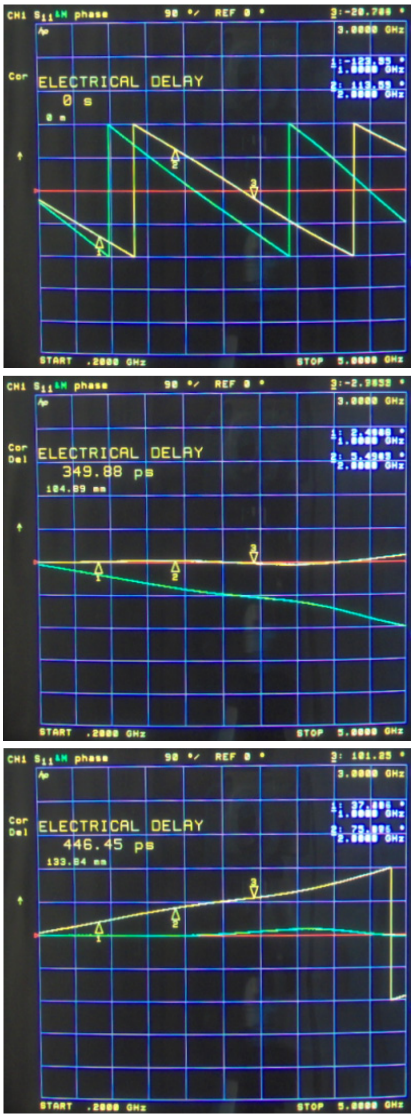


Figure 6 Phase value of the electrical delay corresponding to the reference plane shift to the open end for the two lines (active trace line L_1, memory trace line L_2)

The phase difference at different values of the frequency can be read directly from the VNA. Inserting it in eq. 3, we can find the value of *n*.

This method has the advantage of eliminating the inaccuracy on the line length, since in the determination of *n* only the difference between the two line lengths is present.

Furthermore, it allows to characterize the value of the RI vs. frequency, since measurements are made at single frequency points.

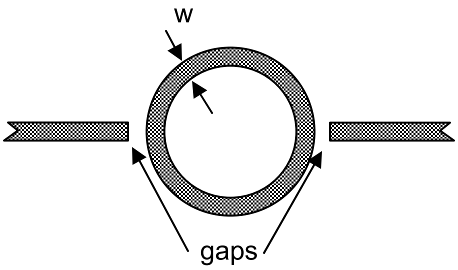
Regarding the measurement in transmission, two lines of different lengths are used. The measured phase difference corresponds to the product between the phase velocity and the difference in lengths. By inverting this expression, the value of n can be obtained.

D. Ring resonator

The method proposed in [3] eliminates completely the inaccuracy due to the position of the reference plane. It makes use of a printed resonant structure by electromagnetic coupling. Referring to the geometry of Fig. 2, the value of the RI is given by:

$$n = \frac{pc}{2f\ell}$$

Where p denotes the order of the resonance, c the speed of light in free space, f the ring resonant frequency and ℓ the principal circumference of the ring resonator.



Measurements can be done both in transmission and in reflection.

The advantage of the method is due to the fact that no phase measurement is needed and, consequently, the VNA can be disposed of.

The values of f and p are determined with the scalar network analyzer (NA). Resonant frequencyies can be defined in different ways: on one hand, they correspond to the frequencies where the imaginary part of the reflection coefficient vanishes. On the other hand, they are given by the minimum value of the reflection coefficient. The two values are slightly different (0.02-0.03 %)

The disadvantage is represented by the fact that the resonant frequencies are influenced by the dimension of the gaps.

III. CONCLUSIONS

Methods for the determination of the refractive index of media were presented. They are based on the measurement of the phase velocity of an RF signal on a microstrip line immersed in the considered media. The circuits required for the proposed measurements are relatively easy to realize in laboratory.

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