Buletinul Ştiinţific al Universităţii "Politehnica" din Timişoara

Seria ELECTRONICĂ şi TELECOMUNICAŢII
TRANSACTIONS on ELECTRONICS and COMMUNICATIONS

Tom 51(65), Fascicola 2, 2006

Lattice MMSE Single User Receiver for Asynchronous DS-CDMA Systems

Constantin Paleologu, Călin Vlădeanu, Andrei A. Enescu[1]

Abstract – This paper considers a lattice Minimum Mean-Squared Error (MMSE) single user adaptive receiver for the asynchronous Direct Sequence – Code Division Multiple Access (DS-CDMA) system. It is based on the Gradient Adaptive Lattice (GAL) algorithm. Since the lattice predictor orthogonalizes the input signals this algorithm achieves a faster convergence rate than the transversal counterpart, the Least Mean Square (LMS) adaptive algorithm, paying with an increased computational complexity. Superior performances are obtained by adapting the tap weights several times during each bit interval.
Keywords: DS-CDMA, adaptive filter, GAL, LMS.

I. INTRODUCTION

There are a lot of mobile communications systems that employ the CDMA (Code Division Multiple Access) technique, where the users transmit simultaneously within the same bandwidth by means of different code sequences. CDMA technique has been found to be attractive because of such characteristics as potential capacity increases over competing multiple access methods, anti-multipath capabilities, soft capacity, narrow-bandwidth anti-jamming, and soft handoff.

In Direct Sequence CDMA (DS-CDMA) systems [1], the conventional matched filter receiver distinguishes each user's signal by correlating the received multi-user signal with the corresponding signature waveform. The data symbol decision for each user is affected by Multiple-Access Interference (MAI) from other users and by channel distortions. Hence, the conventional matched filter receiver performances are limited by its original purpose. It was designed to be optimum only for a single user channel where no MAI is present, and to be optimum for a perfect power control, so it suffers from the near-far problem.

Multi-user receivers have been proposed to overcome the inherent limitations of the conventional matched filter receiver. The use of these multi-user receivers has shown to improve system's performance, and enhance its capacity relative to the conventional matched filter detection. Unfortunately, most of these multi-user detectors require complete system information on all users [1].

Implementations of adaptive Minimum Mean-Squared Error (MMSE) receivers in DS-CDMA systems have been analyzed in [2] and [3]. The principle of the adaptive MMSE receivers consists of a single user detector that works only with the bit sequence of that user. In this case the detection process is done in a bit by bit manner, and the final decision is taken for a single bit interval from the received signal. The complexity of an adaptive MMSE receiver is slightly higher than that of a conventional receiver, but with superior performance [2]-[5]. Besides its facile implementation the adaptive MMSE receiver has the advantage that it needs no supplementary information during the detection process, as compared to the conventional matched-filter receiver.

The adaptive algorithms used for MMSE receivers can be divided into two major categories [6], [7]. The first one contains the algorithms based on mean square error minimization, whose representative member is the Least Mean Square (LMS) algorithm. The second category of algorithms uses an optimization procedure in the least squares (LS) sense, and its representative is the Recursive Least Squares (RLS) algorithm. The transversal LMS algorithm with its simple implementation suffers from slow convergence, which implies long training overhead with low system throughput. On the other hand, LS algorithms such as RLS offer faster convergence rate and tracking capability than the LMS algorithm. This performance improvement of the RLS over the LMS is achieved at the expense of the large computational complexity.

Lattice structures have also been considered for this type of applications [8], [9]. Since the lattice predictor orthogonalizes the input signals, the gradient adaptation algorithms using this structure are less dependent on the eigenvalue spread of the input signal and may converge faster than their transversal counterparts. The computational complexity of the Gradient Adaptive Lattice (GAL) algorithm [6] is between transversal LMS and RLS algorithms. In addition, several simulation examples and also numerical comparison of the analytical results have shown that adaptive lattice filters have better numerical properties than their transversal

[1] University "Politehnica" of Bucharest, Faculty of Electronics, Telecommunications and Information Technology, 1-3, Iuliu Maniu Bvd., Bucharest, Romania, e-mail: pale@comm.pub.ro, calin@comm.pub.ro, aenescu@comm.pub.ro

BUPT

counterparts [10], [11]. Moreover, stage-to-stage modularity of the lattice structure has benefits for efficient hardware implementations.

In this paper we compare the performances of a lattice MMSE single user adaptive receiver based on GAL algorithm for the asynchronous DS-CDMA system with a transversal counterpart based on LMS algorithm.

The paper is organized as follows. In section II we briefly describe the asynchronous DS-CDMA system model, both the transmitter and adaptive receiver parts of the scheme. Section III is focused on the adaptive receiver part of the scheme, revealing in this context the GAL algorithm. The experimental results are presented in section IV. Finally, section V concludes this work.

II. DS-CDMA SYSTEM MODEL

In the transmitter part of the DS-CDMA system, each user data symbol is modulated using a unique signature waveform $a_i(t)$, with a normalized energy over a data bit interval T, $\int_0^T \|a_i(t)\|^2\, dt = 1$, given by [1]:

$$a_i(t) = \sum_{j=1}^{N} a_i(j) p_c(t - jT_c)\,,\ i = \overline{1,\ K} \qquad (1)$$

where the $a_i(j)$ represents the jth chip of the ith user's code sequence and are assumed to be elements of {-1, +1}, and $p_c(t)$ is the chip pulse waveform defined over the interval [0; T_c) with T_c as the chip duration which is related to the bit duration through the processing gain N, with $T_c=T/N$. K denotes the number of users in the system. In the following analysis we consider Binary Phase Shift Keying (BPSK) modulation for signal transmission.

Then, the ith user transmitted signal is given by

$$s_i(t) = \sqrt{2P_i}\, b_i(t) a_i(t) \cos(\omega_0 t + \theta_i)\,,\ i = \overline{1,\ K} \qquad (2)$$

where P_i is the ith user bit power,

$$b_i(t) = \sum_{m=1}^{N_b} b_i(m) p(t - mT),\ \ b_i(m) \in \{-1,\ +1\} \qquad (3)$$

is the binary data sequence for ith user, N_b is the number of received data bits, $\omega_0 = 2\pi f_0$ and θ_i represent the common carrier pulsation and phase, respectively.

A block diagram of the lattice receiver structure is shown in Fig. 1. After converting the received signal to its baseband form using a down converter, the received signal is given by:

$$\begin{aligned} r(t) &= \left[\sum_{i=1}^{K} s_i(t-\tau_i) + n(t)\right]\cos(\omega_0 t) = \\ &= \sqrt{\frac{P_i}{2}} \sum_{i=1}^{K} b_i(t-\tau_i) a_i(t-\tau_i)\cos(\theta_i) + n(t)\cos(\omega_0 t) \end{aligned} \qquad (4)$$

where $n(t)$ is the two-sided PSD (Power Spectral Density) $N_0/2$ additive white Gaussian noise (AWGN). The asynchronous DS-CDMA system consists of random initial phases of the carrier $0 \le \theta_i < 2\pi$ and random propagation delays $0 \le \tau_i < T$ for all the users $i = \overline{1,\ K}$. There is no loss of generality to assume that $\theta_k = 0$ and $\tau_k = 0$ for the desired user k, and to consider only $0 \le \tau_i < T$ and $0 \le \theta_i < 2\pi$ for any $i \ne k$ [2].

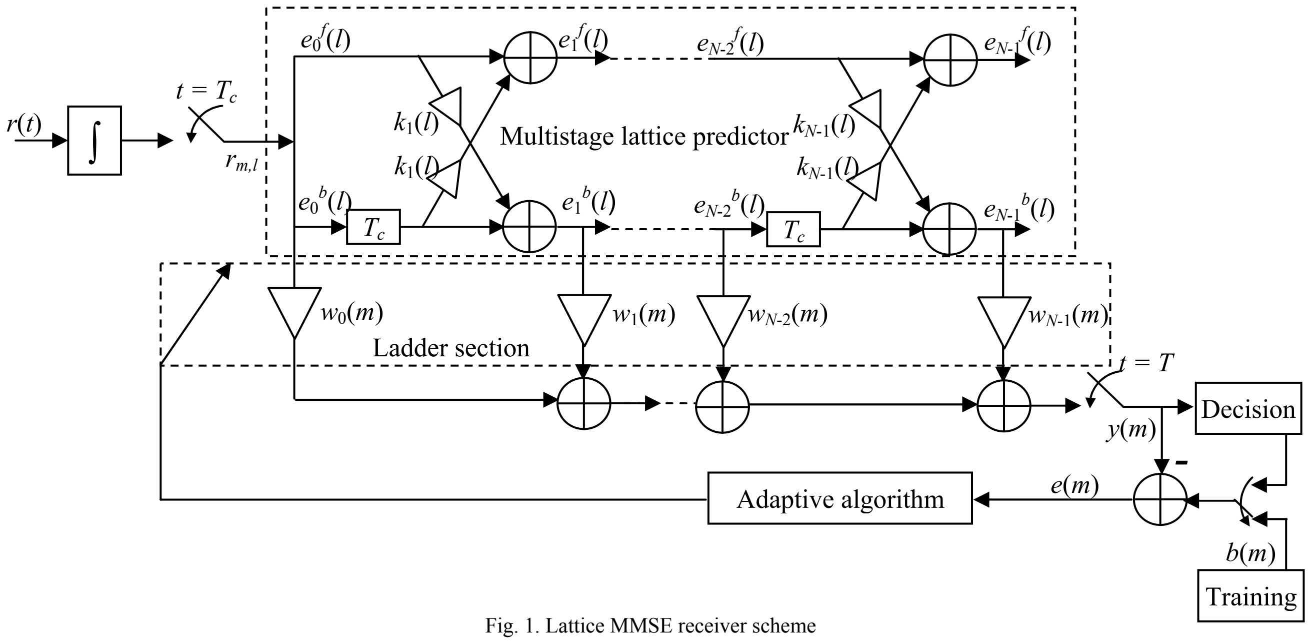

Fig. 1. Lattice MMSE receiver scheme

BUPT

Assuming perfect chip timing at the receiver, the received signal in (4) is passed through a chip-matched filter followed by sampling at the end of each chip interval to give for the *m*th data bit interval:

$$r_{m,\,l} = \int_{mT+lT_c}^{mT+(l+1)T_c} r(t)p(t-lT_c)dt, \quad l=0,\ 1,\ ...,\ N-1 \tag{5}$$

where $p(t)$ is the chip pulse shape, which is taken to be a rectangular pulse with amplitude $1/\sqrt{N}$. Using (5) and taking the *k*th user as the desired one, the output of the chip matched filter after sampling for the *m*th data bit is given by:

$$\begin{aligned} r_{m,l} &= \sqrt{\frac{P_k}{2N}} T_c b_k(m) a_k(l) + \\ &+ \frac{1}{\sqrt{2N}} \sum_{\substack{i=1 \\ i\neq k}}^{K} \sqrt{P_i} \cos\theta_i b_i(m) I_{i,k}(m,l) + n(m,l) \end{aligned} \tag{6}$$

where

$$I_{i,k}(m,l) = \\ = \begin{cases} b_i(m-1)[\varepsilon_i a_i(N-1-N_i+l) + \\ \quad +(T_c-\varepsilon_i)a_i(N-N_i+l)], & 0 \le l \le N_i - 1 \\ b_i(m-1)\varepsilon_i a_i(N-1) + \\ \quad + b_i(m)(T_c-\varepsilon_i)a_i(0), & l = N_i \\ b_i(m)[\varepsilon_i a_i(l-N_i-1) + \\ \quad +(T_c-\varepsilon_i)a_i(l-N_i)], & N_i+1 \le l \le N-1 \end{cases} \tag{7}$$

with

$$\tau_i = N_i T_c + \varepsilon_i\,, \quad 0 \le N_i \le N-1\,, \quad 0 < \varepsilon_i < T_c \tag{8}$$

Let us consider the following vectors:

$$\begin{aligned} \mathbf{r}(m) &= [r_{m,\,0} \quad r_{m,\,1} \quad .... \quad r_{m,\,N-1}]^T \\ \mathbf{a}_k &= [a_k(0), \quad a_k(1) \quad .... \quad a_k(N-1)] \end{aligned} \tag{9}$$

with $r_{m,l}$ given by (6), the vector $\mathbf{a}_k$ represents the binary code sequence for the *k*th user, and the components of the noise $n(m,l)$ vector in (6) consists of independent zero-mean Gaussian random variables with variance $N_0/(2N)$.

In the training mode, the receiver attempts to cancel the MAI and adapts its coefficients using a short training sequence employing an adaptive algorithm. After training is acquired, the receiver switches to the decision-directed mode and continues to adapt and track channel variations [2]-[4].

III. GAL ALGORITHM FOR MMSE RECEIVER

During the training mode, the filter tap weights are adjusted every transmitted bit interval. The receiver forms an error signal proportional to the difference between the filter output and the known reference signal. This error signal is then used to adjust the filter tap weights using the adaptive algorithm. This process is repeated for every received bit until steady-state convergence is reached.

The $(N-1)$-th-order lattice predictor is specified by the recursive equations

$$e_p^f(l) = e_{p-1}^f(l) + k_p^*(l) e_{p-1}^b(l-1) \tag{10}$$

$$e_p^b(l) = e_{p-1}^b(l-1) + k_p(l) e_{p-1}^f(l) \tag{11}$$

where $p = 1, 2, \ldots, N-1$. We denoted by $e_p^f(l)$ the forward prediction error, by $e_p^b(l)$ the backward prediction error, and by $k_p(l)$ the reflection coefficient at the *p*th stage and chip-time *l*. The zeroth-order prediction errors are given by

$$e_0^f(l) = e_0^b(l) = r_{m,l} \tag{12}$$

The cost function used for the estimation of $k_p(l)$ is

$$J_p = \frac{1}{2} E\left\{ \left| e_p^f(l) \right|^2 + \left| e_p^b(l) \right|^2 \right\} \tag{13}$$

where E is the statistical expectation operator [6]. Substituting equations (10) and (11) into equation (13), differentiating the cost function J_p with respect to the complex-valued reflection coefficient $k_p(l)$ and then putting the gradient equal to zero, the optimum value of the reflection coefficient for which the cost function J_p is minimum results

$$k_p^{opt} = -\frac{2E\left\{ e_{p-1}^b(l-1) e_{p-1}^{f*}(l) \right\}}{E\left\{ \left| e_{p-1}^f(l) \right|^2 + \left| e_{p-1}^b(l-1) \right|^2 \right\}} \tag{14}$$

Assuming that the input signal is ergodic the expectations could be substituted by time averages, resulting the Burg estimate for the reflection coefficient k_p^{opt} for stage *p* in the lattice predictor:

$$k_p(l) = -\frac{2\sum_{q=1}^{l} e_{p-1}^b(q-1) e_{p-1}^{f*}(q)}{\sum_{q=1}^{l} \left[\left| e_{p-1}^f(q) \right|^2 + \left| e_{p-1}^b(q-1) \right|^2 \right]} \tag{15}$$

BUPT

Let us denoted by $W_{p-1}(l)$ the total energy of both the forward and backward prediction errors at the input of the pth lattice stage, measured up to and including time l, and expressed it as:

$$W_{p-1}(l)=\sum_{q=1}^{l}\left[\left|e_{p-1}^{f}(q)\right|^{2}+\left|e_{p-1}^{b}(q-1)\right|^{2}\right]= \\ =W_{p-1}(l-1)+\left|e_{p-1}^{f}(l)\right|^{2}+\left|e_{p-1}^{b}(l-1)\right|^{2} \quad (16)$$

It can be demonstrated [6] that the GAL algorithm updates the reflection coefficients using

$$k_p(l)=k_p(l-1)-\frac{\mu}{W_{p-1}(l)}\cdot \\ \cdot\left[e_p^{f*}(l)e_{p-1}^{b}(l-1)+e_p^{b*}(l)e_{p-1}^{f}(l)\right] \quad (17)$$

where μ is a constant controlling the convergence of the algorithm. The use of the time-varying step-size parameter $\mu/W_{p-1}(l)$ in the update equation (8) for the reflection coefficient $k_p(l)$ introduces a form of normalization similar to that in the Normalized LMS (NLMS) algorithm [6], [7]. For a well-behaved convergence of the GAL algorithm, it is recommended that we set $\mu<0.1$ [6].

In practice, a minor modification is made to the energy estimator of equation (16) by writing it in the form of a single-pole average of squared data:

$$W_{p-1}(l)=\beta W_{p-1}(l-1)+(1-\beta)\cdot \\ \cdot\left[\left|e_{p-1}^{f}(l)\right|^{2}+\left|e_{p-1}^{b}(l-1)\right|^{2}\right] \quad (18)$$

where $0<\beta<1$. The introduction of parameter β in equation (18) provides the GAL algorithm with a finite memory, which helps it to deal better with statistical variations when operating in a nonstationary environment. As reported in [10] and demonstrated in [12], the way to choose β is

$$\beta=1-\mu \quad (19)$$

As depicted in Fig. 1, the basic structure for the estimation of the user desired response $b(m)$, is based on a multistage lattice predictor that performs both forward and backward predictions, and an adaptive ladder section. We have an input column vector of the backward prediction errors

$$\mathbf{e}_N^b(m)=[e_0^b(m),e_1^b(m),\ldots,e_{N-1}^b(m)]^T \quad (20)$$

and a corresponding column vector $\mathbf{w}(m)$ representing the N coefficient vector the adaptive filter weights:

$$\mathbf{w}(m)=[w_0(m),w_1(m),....,w_{N-1}(m)]^T \quad (21)$$

where the symbol m denotes the discrete time index of the data bit sequence. The output signal $y(m)$ will be an estimate of $b(m)$. For the estimation of $\mathbf{w}(m)$, we may use a stochastic-gradient approach. The discrete output signal $y(m)$ is given by:

$$y(m)=\sum_{l=0}^{N-1}w_l(m)e_l^b(m) \quad (22)$$

Using vector notation, (22) can be written as:

$$y(m)=\mathbf{w}^T(m)\mathbf{e}_N^b(m) \quad (23)$$

The receiver forms an error signal $e(m)$,

$$e(m)=b(m)-y(m) \quad (24)$$

and a new filter tap weight vector is estimated according to:

$$\mathbf{w}(m+1)=\mathbf{w}(m)+\tilde{\mu}e(m)\mathbf{e}_N^b(m) \quad (25)$$

The parameter $\tilde{\mu}$ in (25) is the ladder structure adaptation step size chosen to optimize both the convergence rate and the mean squared error.

Summarized, we will use equation equations (10), (11), (18) and (17) for the lattice predictor part of the scheme and equations (23)-(25) for the ladder section. Comparative with the its transversal counterpart based on LMS algorithm, the lattice MMSE receiver implies an increased computational complexity due to the multistage lattice predictor. The classical transversal receiver is based only on the equations (23)-(25), where we have to replace $\mathbf{e}_N^b(m)$ by $\mathbf{r}$(m) (see (9)). Nevertheless, due to the fact that the lattice predictor orthogonalizes the input signals, a faster convergence rate is expected.

A solution to increase the overall performances is to adjust the filter tap weights iteratively several times every transmitted bit interval [4], [5]. The error obtained during the Gth iteration of the mth data bit is used by the algorithm in the first iteration of the (m+1)th data bit. When a new data bit is received, the filter tap weights are adapted in the same manner as presented, with the initial condition given by

$$\mathbf{w}^{(0)}(m+1)=\mathbf{w}^{(G)}(m) \quad (26)$$

where $\mathbf{w}^{(0)}(m+1)$ and $\mathbf{w}^{(G)}(m)$ represent the initial tap weights at the (m+1)th received bit, and the final tap weights at time index m, respectively. It is obvious that this process will increase the computational complexity, as well as the speed requirements for the adaptive filter.

BUPT

IV. SIMULATION RESULTS

The asynchronous DS-CDMA system using the lattice MMSE receiver based on GAL algorithm was tested using MATLAB programming environment. It was compared with its transversal counterpart based on LMS algorithm. A binary-phase shift keying transmission in a training mode scenario was considered. The simulation parameters were fixed as follows: the processing gain N = 32, the number of users K = 64 and the signal-to-noise ratio (SNR) is 15 dB. The mean-squared error (MSE) was estimated by averaging over 100 independent trials. The convergence results are presented in Fig. 2.

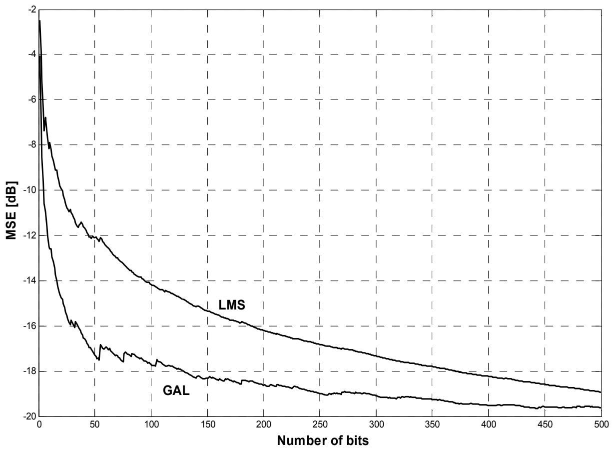

Fig. 2. Convergence of the adaptive receivers

The superior convergence rate achieved by the GAL algorithm as compared to the conventional LMS algorithm can be observed. This can be explained by the fact that the lattice predictor orthogonalizes the input signals. Hence, the gradient adaptation algorithm using this structure is less dependent on the eigenvalue spread of the input signal. In Fig. 3 the mean autocorrelation function is depicted for both the output signal from the chip-matched filter receiver $\mathbf{r}(m)$ (used as the direct input for the transversal LMS receiver) and the sequence of backward prediction errors $\mathbf{e}_N^b(m)$ (the input for the ladder section of the GAL receiver).

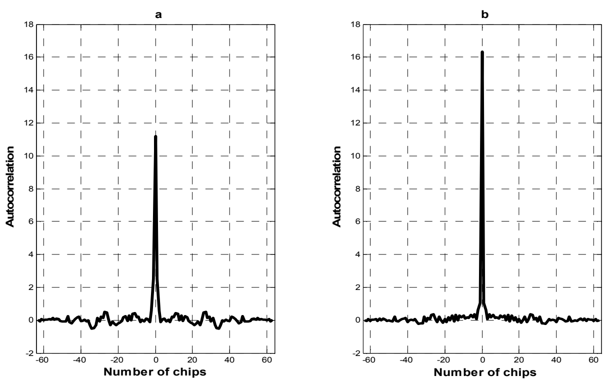

Fig. 3. Autocorrelation functions for:
(a) $\mathbf{r}(m)$ - LMS input; (b) $\mathbf{e}^b_N(m)$ - GAL ladder section input

It is obvious from Fig. 3 that the the input of the GAL ladder section has a higher variance as compared to the LMS input sequence.

As it was mentioned in the end of section III, superior performances are obtained by adapting the tap weights several times during each bit interval. A second set of simulations is dedicated to the proof of this aspect. The adaptive algorithms are iterated for 4 times each data bit. The results are presented in Fig. 4 and Fig. 5.

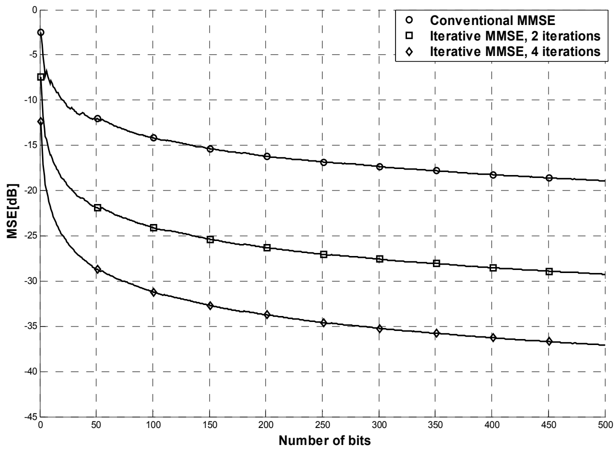

Fig. 4. Convergence of the iterative LMS receiver

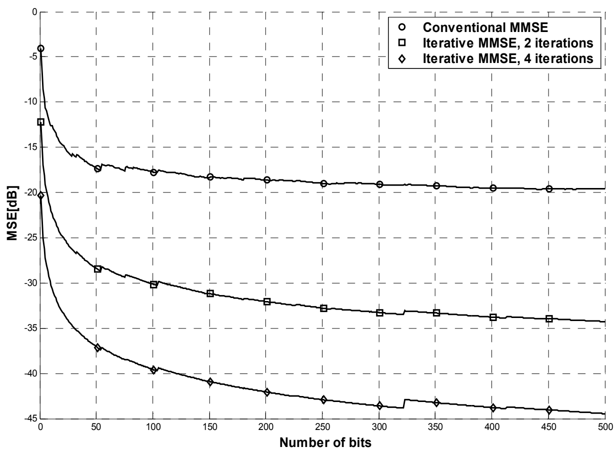

Fig. 5. Convergence of the iterative GAL receiver

In both cases the MSE is decreased every new iteration. Comparing these last two figures it can be also noticed that the GAL algorithm outperforms the LMS algorithm.

V. REMARKS

The lattice MMSE receiver considered in this paper improves the asynchronous DS-CDMA system performances over the classical transversal receiver. The lattice predictor orthogonalizes the input signals, so that the GAL algorithm using this structure is less dependent on the eigenvalue spread of the input signal and may converge faster than their transversal counterpart, the LMS algorithm. As a practical consequence, the lattice receiver will require a shorter

BUPT

training sequence. Superior performances are obtained by adapting the tap weights several times during each bit interval, in order to decrease MSE every new iteration. This decrease offers a faster training mode for the receiver, thus improving the useful bit rate.

As a consequence, the receiver designing procedure may consider one of these two enhancements: to shorten the training sequence for maintaining the same MAI in the system or to strongly reduce the MAI by keeping the same length of the training sequence.

Nevertheless, the systems performances are evaluated by means of MSE. A true performance parameter for the DS-CDMA system is the mean Bit Error Rate (BER). An analytical estimation of BER for this MMSE iterative receiver will be considered in perspective.

REFERENCES

[1] S. Glisic, B. Vucetic, *CDMA Systems for Wireless Communications*, Artech House, 1997.

[2] S. Miller, "An Adaptive Direct-Sequence Code-Division Multiple-Access Receiver for Multi-user Interference Rejection", *IEEE Transactions On Communications*, vol. 43, pp. 1746-1755, Apr. 1995.

[3] P. Rapajic and B. Vucetic, "Adaptive Receiver Structures for Asynchronous CDMA Systems", *IEEE Journal on Selected Areas in Communications*, vol. 12, no. 4, pp. 685-697, May 1994.

[4] W. Hamouda, P. McLane, "A Fast Adaptive Algorithm for MMSE Receivers in DS-CDMA Systems", *IEEE Sign Proc. Letters*, vol. 11, no. 2, pp. 86-89, Feb. 2004.

[5] C. Vlădeanu, C. Paleologu, "MMSE single user iterative receiver for asynchronous DS-CDMA systems", *Proc. of IEEE Int. Conf. Communications 2006*, Bucharest, 2006, pp. 227-230.

[6] S. Haykin, *Adaptive Filter Theory*, fourth edition. Prentice Hall Upper Saddle River, NJ, 2002.

[7] S. Ciochină, C. Negrescu, *Sisteme adaptive*, Ed. Tehnică, Bucharest, 1999.

[8] J. Wang, V. Prahatheesan, "Adaptive Lattice Filters for CDMA Overlay", *IEEE Trans. on Communications*, vol. 48, no. 5, May 2000, pp. 820-828.

[9] F. Takawira, "Adaptive Lattice Filters for Narrowband Interference Rejection in DS Spread Spectrum Systems", *Proc. IEEE South African Symp. Communications and Signal Processing*, 1994.

[10] V. J. Mathews, Z. Xie, "Fixed-Point Error Analysis of Stochastic Gradient Adaptive Lattice Filters", *IEEE Trans. on Acoustics, Speech and Signal Processing*, vol. 38, no. 1, January 1990, pp. 70-79.

[11] R. C. North, J. R. Zeidler, W. H. Ku, T. R. Albert, "A Floating-Point Arithmetic Error Analysis of Direct and Indirect Coefficient Updating Techniques for Adaptive Lattice Filters", *IEEE Trans. on Signal Processing*, vol. 41, no. 5, May 1993, pp. 1809-1823.

[12] C. Paleologu, S. Ciochină, A.A. Enescu, "Modified GAL Algorithm Suitable for DSP Implementation", *Proc. of IEEE Int. Symp. ETc 2002*, vol. 1, Timişoara, 2002, pp. 2-7.

BUPT