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Adapting a Normalized Gradient Subspace Algorithm to Real-Valued Data Model

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Abstract – A new gradient approach to adaptive subspace-based frequency estimation of multiple real valued sine waves is considered in this paper. The new approach proposed here combines the normalized gradient subspace tracking technique based on Oja learning rule - NOOja (for the signal subspace update) with the ESPRIT-like frequency estimation of real-valued sinusoids (for frequency values retrieval). Consequently, a new adaptive subspace-tracking algorithm for frequency estimation is proposed. The method proposed brings a significant reduction in arithmetical complexity at the same level of accuracy. The algorithm is tested in numerical simulations and compared to complex-valued NOja method.

Keywords: subspace tracking, frequency estimation, real-valued data, R-ESPRIT, NOja

I. INTRODUCTION

Adaptive subspace tracking for determining time-varying frequencies of sine wave carriers is a research field still under study. Not only old techniques have been optimized [6], [7], but also new algorithms have been developed in order to improve the accuracy of the methods or to decrease the computational burden [2], [8].

However, traditional methods present the major drawback of assuming that the data are complex-valued and this implies additional computational effort. All super-resolution subspace block methods (MUSIC, ESPRIT etc.) are based on a complex-valued signal model, as they have initially been designed for array processing [4]. Only recently Mahata and Söderström developed an ESPRIT-like method to estimate the real-valued sinusoidal frequencies [1], [9]. This new non-iterative method, called R-ESPRIT by the authors, is based on a real-valued signal model and brings a spectacular reduction in the number of operations required to compute the frequency estimates.

It is then natural to think about adaptive methods able to take advantage of this much lower complexity. In the present paper we have made a further step to the work presented in [2] in the context of projection approximation subspace tracking and adapted the well-known normalized orthogonal gradient subspace-tracking algorithm based on Oja learning rule NOOja to the real-valued signal model.

Similar to the method presented in [2] for PAST algorithm, the subspace tracking-type NOOja method is modified for applying R-ESPRIT for real sinusoids retrieval. We will name the new algorithm R-NOOja. From the author's knowledge, the new method has never been published before.

We will compare the performances and the complexity of the newly derived algorithm with the NOOja method based on the complex-valued data model.

II. SIGNAL MODEL

The signal model is presented in [2]. We will briefly review it here for present paper consistency.

The input signal consists in a number r of sinusoidal signals that may well be sine wave carriers, embedded in white Gaussian noise:

$$x(t)=\sum_{k=1}^{r} s_k \sin(t\omega_k+\phi_k)+n(t)\,, \qquad (1)$$

where s_k is the amplitude, ω_k is the angular frequency of the of the k^{th} sinusoid and $n(t)$ represents the corrupting additive zero-mean white noise. The phases $\{\phi_k\}_{k=1}^{r}$ are random variables uniformly distributed in the $[-\pi,\pi]$ interval.

The compact subspace representation dedicated for real valued sinusoids differs from the classical complex-valued signal model [1]. We have to obtain an alternative snapshot vector so that its noise-free part lies in a subspace of dimension r. To that aim, we will introduce the following input vectors:

$$\mathbf{x}_c(t)\overset{\Delta}{=}[x(t)\quad \ldots \quad x(t+n-1)]^T \qquad (2)$$

$$\mathbf{x}_b(t)\overset{\Delta}{=}[x(t-1)\quad \ldots \quad x(t-n)]^T \qquad (3)$$

$$\mathbf{x}_r(t)\overset{\Delta}{=}\frac{1}{2}\{\mathbf{x}_c(t)+\mathbf{x}_b(t)\} \qquad (4)$$

where the snapshot vector dimension $n>2r$. From the above definitions we obtain

$$\mathbf{x}_r(t)=\mathbf{A}_r\mathbf{s}_r(t)+\mathbf{n}_r(t) \qquad (5)$$

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where $\mathbf{s}_r(t)$ is an $r\times 1$ vector given by

$$\mathbf{s}_r(t)=\begin{bmatrix} a_1\cos\{\omega_1 t+\phi_1^+\} \\ \vdots \\ a_r\cos\{\omega_r t+\phi_r^+\}\end{bmatrix} \quad (6)$$

where $\phi_k^+=\phi_k-(1/2)\omega_k$ for $1\le k\le r$. $\mathbf{A}_r$ is an $n\times r$ matrix given by

$$\mathbf{A}_r=\begin{bmatrix} \cos\left(\frac{\omega_1}{2}\right) & \cdots & \cos\left(\frac{\omega_r}{2}\right) \\ \cos\left(\frac{3\omega_1}{2}\right) & \cdots & \cos\left(\frac{3\omega_r}{2}\right) \\ \vdots & \ddots & \vdots \\ \cos\left\{\left(n-\frac{1}{2}\right)\omega_1\right\} & \cdots & \cos\left\{\left(n-\frac{1}{2}\right)\omega_r\right\}\end{bmatrix}. \quad (7)$$

The noise snapshot vector $\mathbf{n}_r(t)$ in this modified model is given by

$$\mathbf{n}_r(t)=\frac{1}{2}\{\mathbf{n}_c(t)+\mathbf{n}_b(t)\} \quad (8)$$

where

$$\mathbf{n}_c(t)\overset{\Delta}{=}[n(t) \quad \dots \quad n(t+n-1)]^T \quad (9)$$

$$\mathbf{n}_b(t)\overset{\Delta}{=}[n(t-1) \quad \dots \quad n(t-n)]^T \quad (10)$$

One can show [1] that $\mathbf{A}_r$ is a full column rank matrix. The important fact here is that the noise-free part of $\mathbf{x}_r(t)$ lies in an r-dimensional subspace that is different from the complex-valued data model, where the dimension of the signal subspace is $2r$.

Further on, let us introduce

$$\mathbf{P}_r\overset{\Delta}{=}E\{\mathbf{s}_r(t)\mathbf{s}_r^T(t)\}. \quad (11)$$

The noise vectors $\mathbf{n}_c(t)$ and $\mathbf{n}_b(t)$ are random vectors, mutually independent, with $E\{\mathbf{n}_r(t)\mathbf{n}_r^T(t)\}=(\sigma^2/2)\mathbf{I}_n$ where σ^2 is the noise variance. We obviously have that

$$\mathbf{R}_r\overset{\Delta}{=}E\{\mathbf{x}_r(t)\mathbf{x}_r^T(t)\}=\mathbf{A}_r\mathbf{P}_r\mathbf{A}_r^T+\frac{\sigma^2}{2}\mathbf{I}_n. \quad (12)$$

We may then consider the eigenvalue decomposition

$$\mathbf{R}_r=\mathbf{S}_r\mathbf{\Lambda}_r\mathbf{S}_r^T+\mathbf{G}_r\mathbf{\Sigma}_r\mathbf{G}_r^T \quad (13)$$

where $\mathbf{\Lambda}_r$ is an $r\times r$ diagonal matrix containing the r dominant eigenvalues of $\mathbf{R}_r$ on the diagonal. The $n\times r$ matrix $\mathbf{S}_r$ is composed of the corresponding left eigenvectors. In the same perspective, $\mathbf{\Sigma}_r$ is a diagonal matrix containing the remaining $n-r$ eigenvalues of $\mathbf{R}_r$. The $n\times(n-r)$ matrix $\mathbf{G}_r$ is composed of the corresponding left eigenvectors. The columns of $\mathbf{G}_r$ are orthogonal to those of $\mathbf{S}_r$.

One can show (see [1]) that

$$\mathbf{S}_r=\mathbf{A}_r\mathbf{C}_r \quad (14)$$

where

$$\mathbf{C}_r=\mathbf{P}\mathbf{A}_r^T\mathbf{S}_r\left\{\mathbf{\Lambda}_r-\frac{\sigma^2}{2}\mathbf{I}_n\right\}^{-1}. \quad (15)$$

The columns of $\mathbf{S}_r$ form an orthonormal basis of the column space of $\mathbf{A}_r$. The idea is to adaptively obtain an estimate of $\mathbf{S}_r$ from the data via the normalized orthogonal gradient adaptive subspace tracking method NOOja, which will then be processed to obtain the frequency estimates.

III. ALGORITHMS

A. *R-Esprit*

The R-ESPRIT algorithm is an ESPRIT-like estimation method of real-valued sinusoidal frequencies. The algorithm has been proposed and has been presented in detail in [1] and [9]. We will resume as in [2] the main aspects of this method as it represents a key factor in developing our new adaptive method. R-ESPRIT relies on the signal model presented in Section 2 of the present paper.

The basic idea is to make use of two $(n-2)\times n$ Toeplitz matrices

$$\mathbf{T}_r^{(1)}=\begin{bmatrix} 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & 0 & 1 & 0\end{bmatrix} \quad (16)$$

$$\mathbf{T}_r^{(2)}=\frac{1}{2}\begin{bmatrix} 1 & 0 & 1 & 0 & \cdots & 0 \\ 0 & 1 & 0 & 1 & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & 1 & 0 & 1\end{bmatrix}. \quad (17)$$

From the definition of matrix (see relation (7)) the following identity holds:

$$\mathbf{T}_r^{(2)}\mathbf{A}_r=\mathbf{T}_r^{(1)}\mathbf{A}_r D_r \quad (18)$$

where D_r is the following diagonal matrix:

$$D_r=\mathrm{diag}\{\cos(\omega_1), \quad \dots \quad ,\cos(\omega_n)\}. \quad (19)$$

Let us also introduce the following matrix

$$\mathbf{\Phi}_r=\mathbf{C}_r^{-1}D_r\mathbf{C}_r \quad . \quad (20)$$

Then, the algorithm may be derived as follows:

a) It is first required to estimate $\hat{\mathbf{S}}_r$ from the input data.

b) This estimate will be used in estimating $\hat{\mathbf{\Phi}}_r$, from (14) and (18), as

$$\hat{\mathbf{\Phi}}_r=(\mathbf{T}_r^{(1)}\hat{\mathbf{S}}_r)^{-1}\mathbf{T}_r^{(2)}\hat{\mathbf{S}}_r. \quad (21)$$

c) The eigendecomposition of $\hat{\mathbf{\Phi}}_r$ will lead us to $\hat{D}_r$, following equation (20).

d) Knowing $\hat{D}_r$, frequency values easily result from:

$$\omega_k=\cos^{-1}\{\hat{D}_r(k,k)\} \qquad k=1,\dots,r \quad (22)$$

This finally gives the frequency estimates. The dimension of the signal subspace is reduced to r from $2r$ i.e. the case of traditional ESPRIT method for r real-valued sine waves.

B. *The R-NOOja algorithm*

In this chapter we will derive a novel adaptive method for estimating the signal subspace $\hat{\mathbf{S}}_r$ from the input data. This algorithm is based on the NOOja adaptive method proposed in [7] and modified for the real data model presented in Section 2 of this paper. We will therefore refer to this algorithm as R-NOOja. From the authors' knowledge, this method has never been published in this form before.

Let $\mathbf{x}_r \in \boldsymbol{R}^n$ be the input data vector at time *t* defined as in relation (5), with the correlation matrix $\mathbf{R}_r = E\{\mathbf{x}_r \mathbf{x}_r^H\}$.

Note here that *r* represents the number of real sinusoids, which is half the number of complex sinusoids used in traditional adaptive methods for tracking frequencies of real sine waves. This is the first level of reduction in arithmetical complexity. The second level comes from the use of R-ESPRIT technique, a much simpler method adapted for real sinusoids environments, instead of ESPRIT, traditionally proposed for complex-valued signal environments.

We are interested to recursively estimate the signal subspace $\hat{\mathbf{S}}_r$, therefore to compute the signal subspace estimate at the time instant *t* from the subspace estimate at *t* – 1 and the new arriving sample vector $\mathbf{x}_r$.

As in [7] we know that if we consider the following cost function:

$$\begin{aligned} J(\mathbf{W}_r(t)) &= E\{\|\mathbf{x}_r - \mathbf{W}_r(t)\mathbf{W}_r^T(t)\mathbf{x}_r\|^2\} \\ &= \mathrm{tr}\left[\mathbf{R}_r(t)\right] - 2\,\mathrm{tr}\left[\mathbf{W}_r^T(t)\mathbf{R}_r(t)\mathbf{W}_r^T(t)\right] \\ &\quad + \mathrm{tr}\left[\mathbf{W}_r^T(t)\mathbf{R}_r(t)\mathbf{W}_r(t)\mathbf{W}_r^T(t)\mathbf{W}_r(t)\right] \end{aligned} \quad (23)$$

where $\mathbf{W}_r(t)$ is a real-valued $n \times r$ matrix, then following the theory in [3] and [5], we can prove that the matrix $\mathbf{W}_r(t) \in \boldsymbol{R}^{n \times r}\ (r < n)$ minimizing $J(\mathbf{W}_r(t))$ is a good estimate for the signal subspace $\hat{\mathbf{S}}_r(t)$ of the correlation matrix $\mathbf{R}_r(t)$.

We can compute the gradient of the cost function

$$\begin{aligned} \nabla J(\mathbf{W}_r(t)) &= [-2\mathbf{R}_r(t) + \mathbf{R}_r(t)\mathbf{W}_r(t)\mathbf{W}_r^T(t) \\ &\quad + \mathbf{W}_r(t)\mathbf{W}_r^T(t)\mathbf{R}_r(t)]\mathbf{W}_r(t) \end{aligned} \quad (24)$$

and we can write the signal subspace update as

$$\begin{aligned} \mathbf{W}_r(t) &= \mathbf{W}_r(t-1) - \mu[-2\hat{\mathbf{R}}_r(t) + \hat{\mathbf{R}}_r(t)\mathbf{W}_r(t-1)\mathbf{W}_r^T(t-1) \\ &\quad + \mathbf{W}_r(t-1)\mathbf{W}_r^T(t-1)\hat{\mathbf{R}}_r(t)]\mathbf{W}_r(t-1) \end{aligned} \quad (25)$$

where $\mu > 0$ represents the adaptation step and $\hat{\mathbf{R}}(t)$ represents the estimate of the correlation matrix $\mathbf{R}$ at time instant *t*.

The simplest method to estimate matrix $\mathbf{R}(t)$ is to consider the instantaneous estimate $\hat{\mathbf{R}}(t) = \mathbf{x}(t)\mathbf{x}^H(t)$ according to LMS method from adaptive filtering. The following recursive formulas for updating the signal subspace result:

$$\mathbf{y}_r(t) = \mathbf{W}_r^T(t-1)\mathbf{x}_r(t) \quad (26)$$

$$\begin{aligned} \mathbf{W}_r(t) &= \mathbf{W}_r(t-1) + \mu[2\mathbf{x}_r(t)\mathbf{y}_r^T(t) \\ &\quad - \mathbf{x}_r(t)\mathbf{y}_r^T(t)\mathbf{W}_r^T(t-1)\mathbf{W}_r(t-1) \\ &\quad - \mathbf{W}_r(t-1)\mathbf{y}_r(t)\mathbf{y}_r^T(t)] \end{aligned} \quad (27)$$

Further on, we see that we may approximate

$$\mathbf{W}_r^T(t-1)\mathbf{W}_r(t-1) \cong \mathbf{I}_r \quad (28)$$

where $\mathbf{I}_r$ is the $r \times r$ identity matrix. Thus, we obtain a simplified version of the gradient method that represents, in fact, the Oja learning rule:

$$\mathbf{W}_r(t) = \mathbf{W}_r(t-1) + \mu[\mathbf{x}_r(t) - \mathbf{W}_r(t-1)\mathbf{y}_r(t)]\mathbf{y}_r^T(t). \quad (29)$$

Approximation (28) is based on the observation that, for stationary signals, $\mathbf{W}_r(t)$ will converge towards a matrix with orthonormal columns (if $\mu = \mu(t) \xrightarrow{t\to\infty} 0$), or almost orthonormal (if μ is small and constant).

In order to obtain the normalized orthogonal version of Oja algorithm we will add an orthonormalization step of the real-valued matrix $\mathbf{W}_r(t)$ and we will write

$$\mathbf{W}_r(t) = \mathbf{W}_r(t)\left(\mathbf{W}_r^T(t)\mathbf{W}_r(t)\right)^{-1/2} \quad (30)$$

where $\left(\mathbf{W}_r^T(t)\mathbf{W}_r(t)\right)^{-1/2}$ denotes the inverse of the square root for the matrix $\left(\mathbf{W}_r^T(t)\mathbf{W}_r(t)\right)$.

As in [7], one can easily show that

$$(\mathbf{W}_r^T(t)\mathbf{W}_r(t))^{-1/2} = \mathbf{I} + \tau(t)\mathbf{y}_r(t)\mathbf{y}_r^T(t) \quad (31)$$

where

$$\tau(t) \stackrel{\Delta}{=} \frac{1}{\|\mathbf{y}_r(t)\|^2}\left(\frac{1}{\sqrt{1 + \mu_{opt}^2(t)\|\mathbf{e}(t)\|^2\|\mathbf{y}_r(t)\|^2}} - 1\right) \quad (32)$$

and

$$\mathbf{e}(t) = \mathbf{x}_r(t) - \mathbf{W}_r(t-1)\mathbf{y}_r(t). \quad (33)$$

Taking into account that $\mathbf{W}_r(t)$ is now orthogonal at each iteration, i.e. $\mathbf{W}_r^T(t)\mathbf{W}_r(t) = \mathbf{I}$, we may write

$$\hat{\nabla} J(\mathbf{W}_r(t)) = -[\mathbf{x}_r(t) - \mathbf{W}_r(t-1)\mathbf{y}_r(t)]\mathbf{y}_r^T(t). \quad (34)$$

The optimal variable stepsize at time instant *t* becomes

$$\hat{\mu}_{opt}(t) = \frac{1}{\|\mathbf{y}_r(t)\|^2 - \|\mathbf{x}_r(t)\|^2} \quad (35)$$

where one can show that $\|\mathbf{y}_r(t)\|^2 - \|\mathbf{x}_r(t)\|^2 \le 0$.

Following the relations (1) to (35) and adapting the method in [7] we can easily derive a new normalized orthogonal subspace tracking gradient algorithm based on Oja learning rule and adapted to real sinusoidal carrier environments. We name this method R-NOOja. We will prove in the next section that the newly derived method performs as well as complex-valued NOOja algorithm at a much less computational burden.

Table 1 briefly presents the subspace tracking R-NOOja algorithm adapted for sinusoidal carriers frequency identification. Here $\mathbf{x}_r(t)$ represents the input vector at time *t*.

Table 1
R-NOOJA ALGORITHM FOR REAL-VALUED FREQUENCY ESTIMATION

r = number of real sinusoids
n = dimension of $\mathbf{x}_r(t)$

$$\mathbf{W}(0) = \begin{bmatrix} \mathbf{I}_r \\ \mathbf{0}_{n-r} \end{bmatrix}$$

FOR t = 1,2, … DO

$$\mathbf{x}_r(t) \overset{\Delta}{=} \frac{1}{2}\{\mathbf{x}_c(t) + \mathbf{x}_b(t)\}$$

$$\mathbf{y}_r(t) = \mathbf{W}_r^T(t-1)\mathbf{x}_r(t)$$

$$\mathbf{z}(t) = \mathbf{W}_r(t-1)\mathbf{y}_r(t)$$

$$\mathbf{e}(t) = \mathbf{x}_r(t) - \mathbf{z}(t)$$

$$\hat{\mu}_{opt}(t) = \frac{-\mu}{\|\mathbf{y}_r(t)\|^2 - \|\mathbf{x}_r(t)\|^2 + \gamma}$$

$$\phi(t) = \frac{1}{\sqrt{1 + \hat{\mu}_{opt}^2(t)\|\mathbf{e}(t)\|^2\|\mathbf{y}_r(t)\|^2}}$$

$$\tau(t) = \frac{\phi(t) - 1}{\|\mathbf{y}_r(t)\|^2}$$

$$\mathbf{p}(t) = -\frac{1}{\hat{\mu}_{opt}(t)}\tau(t)\mathbf{z}(t) + \phi(t)\mathbf{e}(t)$$

$$\mathbf{u}(t) = \frac{\mathbf{p}(t)}{\|\mathbf{p}(t)\|}$$

$$\mathbf{v}(t) = \mathbf{W}_r^T(t-1)\mathbf{u}(t)$$

$$\mathbf{W}_r(t) = \mathbf{W}_r(t-1) - 2\mathbf{u}(t)\mathbf{v}^T(t)$$

$$\hat{\mathbf{S}}_r(t) = \mathbf{W}_r(t)$$

$$\mathbf{f}(t) = \mathrm{R-ESPRIT}(\hat{\mathbf{S}}_r(t))$$

END FOR

Estimated frequencies vector $\mathbf{f}$ is obtained by applying R-ESPRIT method (see section 3.1.) to the orthonormal basis $\mathbf{W}_r$ of signal subspace.

Here μ and γ represent two positive constants ($0 < \mu < 1$) that help in improving the numerical stability of the algorithm [7].

IV. SIMULATION RESULTS

A. *Evaluation of arithmetical complexity*

We evaluate the computational effort for the main loop of each algorithm in order to better compare the two methods R-NOOja and NOOja in the context of real-valued sinusoidal carriers frequency identification. Performance of subspace tracking-type algorithms depends not only on the number r of sinusoids, but also on the dimension n of the input vector $\mathbf{x}_r(t)$.

We obtain the following estimations for the arithmetical complexity of the main loop (where operation means real numbers addition or multiplication):

$$NOOja: \; 18nr + 19n + 24r + 16 \;\; operations / iteration$$

$$R-NOOja: 9nr + 21n + 12r + 16 \;\; operations / iteration$$

Even if both algorithms are $O(nr)$, we can clearly see that R-NOOja algorithm requires fewer operations than NOOja method for computing the update of the signal subspace estimate $\mathbf{W}_r$. Further gain in computational burden comes from the use of R-ESPRIT instead of ESPRIT for the values of the frequency estimates. A detailed comparison of these two block methods from complexity point of view may be found in [9].

From extensive simulations, we may state that the overall computational effort for R-NOOja is only about 40% as compared to complex-valued NOOja for the same input vector dimension. We have checked the results with MATLAB `flops` routine.

B. *Algorithms behavior in stationary environments*

We study the statistical properties of both R-NOOja and NOOja algorithms in stationary environments. We are interested to see if the reduction in arithmetical complexity affects the algorithm performance. We present the results obtained for the two algorithms when retrieving two sinusoids of normalized frequencies $f_1 = 0.1, f_2 = 0.2$, embedded in background white noise. We have considered $\mu = 0.5$ and $\gamma = 10$ for both methods in order to achieve best numerical stability.

We calculate the bias and variance of the estimated frequencies for various signal lengths N and for various signal-to-noise ratios. In each case, we run 100 independent simulations. Each time we compute the Cramer-Rao bound (CRB) to verify the accuracy of the estimates.

Table 2
STATISTICAL RESULTS FOR R-NOOJA (n=2r+5)

N	SNR (dB)	Bias f_1	Var f_1	Bias f_2	Var f_2	CRB
100		**x10^{-4}**	**x10^{-4}**	**x10^{-4}**	**x10^{-4}**	**x10^{-4}**
	0	7.22	61.15	8.16	42.17	3.96
	10	-1.26	15.32	2.05	15.17	1.25
	20	-0.55	4.76	0.46	4.91	0.40
	30	-0.20	1.50	0.12	1.56	0.13
200		**x10^{4}**	**x10^{-4}**	**x10^{-4}**	**x10^{-4}**	**x10^{-5}**
	0	1.79	51.92	2.62	46.67	13.90
	10	-1.46	15.40	0.59	13.45	4.39
	20	-0.62	4.78	-0.030	4.36	1.39
	30	-0.21	1.50	-0.008	1.38	0.44
500		**x10^{-5}**	**x10^{-4}**	**x10^{-5}**	**x10^{-4}**	**x10^{-5}**
	0	116.78	52.68	95.45	49.09	3.50
	10	25.54	14.27	10.59	13.57	1.11
	20	6.84	4.48	1.08	4.24	0.35
	30	2.00	1.42	0.12	1.34	0.11

Table 3

STATISTICAL RESULTS FOR NOOJA (n=2r+5)

N	SNR (dB)	Bias f_1	Var f_1	Bias f_2	Var f_2	CRB
100		**x10^{-4}**	**x10^{-4}**	**x10^{-4}**	**x10^{-4}**	**x10^{-4}**
	0	-8.98	157.03	16.52	90.50	3.96
	10	0.38	20.02	1.71	17.26	1.25
	20	0.60	6.77	0.33	5.66	0.40
	30	0.76	2.17	-0.06	1.81	0.13
200		**x10^{-4}**	**x10^{-4}**	**x10^{-4}**	**x10^{-4}**	**x10^{-5}**
	0	-7.56	64.34	2.06	59.10	13.9
	10	1.57	22.14	-1.16	18.16	4.39
	20	-0.61	7.35	-0.39	5.86	1.39
	30	-0.21	2.33	-0.13	1.84	0.44
500		**x10^{-5}**	**x10^{-4}**	**x10^{-5}**	**x10^{-4}**	**x10^{-5}**
	0	-97.99	65.63	164.5	63.89	3.50
	10	-5.54	22.00	-8.37	20.90	1.11
	20	2.47	6.86	5.11	5.92	0.35
	30	0.92	2.19	1.26	1.90	0.11

Tables 2 and 3 present the statistical performances R-NOOja and NOOja algorithms, respectively. We see that R-NOOja overall performs about the same as NOOja at a much lower arithmetical complexity. We also see that both algorithms converge in less than 100 iterations.

IV. CONCLUSIONS

In the present paper we have moved forward to the work presented in [2] and adapted the normalized gradient subspace tracking technique based on Oja learning rule - NOOja [3], [7] to the real-valued signal model. Thus, we derive another novel gradient subspace method, optimized for tracking real sinusoidal carriers in noise. We name this method R-NOOja. The new algorithm uses the real data model. We compare its performances to the complex-valued NOOja algorithm. We conclude that R-NOOja has about the same performances as NOOja in stationary environments, but at much lower computational effort.

It seems that we can further mitigate the major drawback in the use of subspace tracking-type algorithms, their high arithmetical complexity.

This paper follows-up the authors work in [2] in the field of optimizing adaptive subspace tracking methods like [6] for estimating frequencies of real valued sinusoids in noise. This is also the perspective of our future studies.

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