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Nonlinear Channel Equalization Using Complex Neural Networks

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Abstract This paper offers an overview of complex equalizers, which combine the structure of a linear transversal filter (LTE) with a neural network. There are presented nonlinear channel and nonlinear device models. There are exposed different equalizers architectures and some of the training algorithms. Simulation results are presented and it is made a comparative study of the complex equalizers performances, from the point of view of signals space partition and processing error, in different conditions of noise. Concluding remarks and further developments are discussed.

Keywords: channel equalization, complex neural networks, classification

I. INTRODUCTION

In modern high-speed communications networks, the presence of symbol interference (ISI) is a major impediment of transmission. Nonlinear active or passive devices and the transmission channels themselves introduce nonlinear distortions that affect the signals. Especially the signals with a variable envelope modulation, as for example the quadrature amplitude modulation (MAQ) signals, more efficient in transmission from the spectral point of view, are affected, in phase and in amplitude. To eliminate the distortions of MAQ and phase shift keying (PSK) signals, NN equalizers for complex signals are necessary. Treating the problem of equalization as a problem of signal classification, neural networks (NN) can produce arbitrarily complex decision regions. The complex NN equalizers are straight-forward extensions from the real counterparts [12], obtained by replacing the relevant parameters with complex values. Studies performed during the last decade have established the superiority of neural equalizers comparative to the traditional equalizers, in conditions of high nonlinear distortions and rapidly varying signals.

Various neural equalizers have been developed, mostly combinations between a conventional linear transversal filter (LTE) and a neural network: a LTE and a multilayer perceptron (MLP) [1], [8], a LTE and a radial basis function network (RBF) [2], [3], [4],[12] a LTE and a recurrent neural network (RNN) [10], [16], [18], a functional link equalizer [7] [15], and cellular neural network equalizer [17]. The LTE eliminates the linear distortions, such as ISI, so the NN has to compensate the nonlinearities. Many different nonlinear devices models and channels models have been introduced to simulate real situations, so a unitary comparison between all known equalizers is difficult to be done.

In this paper an overview of complex NN equalizers is presented. The problem of equalization, some nonlinear channels and nonlinear device models are exposed. There are treated different structures and some of the training algorithms. Considering the advantages of RBF NN over the MLP, the problems of RBF are detailed and a new training algorithm is presented. Performance comparison is discussed and some simulation results are given. It is shown that NN equalizers efficiently approximate the optimal decision boundaries, having a very good symbol error rate performance. They outperform the conventional equalizers especially when complicated modulation schemes are used.

II. THE EQUALIZATION PROBLEM

The equalization problem is traditionally viewed as an inverse filter problem. Equalizers are designed to track the time-varying channel distortions by adjusting their coefficients and maintaining a prescribed signal to noise ratio (SNR). Tradeoffs between noise enhancement and channel inversion generally render these techniques suboptimal. An alternative viewpoint is to consider the equalization problem as a pattern classification problem.

The objective of equalization becomes the separation of the received symbols in the output signal space, whose optimal decision region boundaries are generally highly nonlinear. Since neural networks are well known for their ability of performing classification tasks by forming complex nonlinear decision boundaries, neural equalizers have been recently receiving considerable attention. Neural equalizers have shown the potential for significant performance improvements especially in severely nonlinear distorted and rapidly varying signals.

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Fig. 1 represents a model of a communication system.

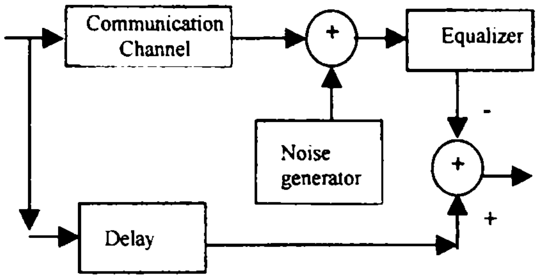

Fig. 1 The scheme of a communication system

If the transmitted signal x is 4 QAM , the input constellation is given by:

$$x(n) = x_R + jx_I = \begin{cases} x^{(1)} = 1 + j \\ x^{(2)} = -1 + j \\ x^{(3)} = 1 - j \\ x^{(4)} = -1 - j \end{cases} \quad (1)$$

The input symbols sequence x(n) is passed through the nonlinear communication channel model and produces the output sequence y(n). The channel output signal is affected by an additive noise, usually white Gaussian, and generates a corrupted signal.

The problem of equalization is to determine an estimation of the input signal using the received signal and the desired delayed signal x(n-d). From the NN point of view, the equalizer has to classify the received signal in one of the four possible classes $P_{m,d}$, according to the input signals:

$$P_{m,d} = \bigcup_{1 \le l \le 4} P_{m,d}(l) \quad (2)$$

or:

$$P_{m,d}(l) = \{ y(n) | x(n-d) = x^{(l)} \} \quad 1 \le l \le 4 \quad (3)$$

III. THE COMMUNICATION CHANNEL MODEL

Nonlinear, active or passive devices and the communication channels themselves introduce distortions that affect the transmitted signals. Fig.2 represents a model of the communication channel that introduces linear (L) and nonlinear distortions (NL).

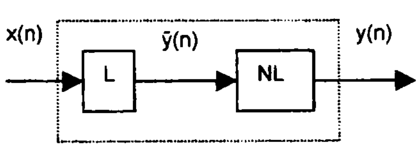

Fig.2 Nonlinear channel model

Usually the linear complex part of the channel is modeled with a transversal filter whose output is given by:

$$\tilde{y}(n) = \sum_{i=0}^{k-1} a_i x(n-i) \quad (4)$$

where a_i are the filter coefficients and k is the order of the filter.

Various models with different linearities and nonlinerities are mentioned in literature. Most of the studies refer to the ones mentioned in that follows. The model suggested in [2] generates the output signal y(n) according to relation:

$$\tilde{y} = (0.34 - 0.27j)x(n) + (0.87 + 0.43j)x(n-1) + \\ + (0.34 - 0.21j)x(n-2) \quad (5)$$

The nonlinear part of the channel is a very strong one and produces at the output:

$$y(n) = \tilde{y}(n) + 0.1[\tilde{y}(n)]^2 + 0.05[\tilde{y}(n)]^3 \quad (6)$$

Another model [3] uses the following equations:

$$\tilde{y} = (0.7409 - 0.7406j)x(n) - (0.8890 - 0.2961j)x(n-1) + \\ + (0.1556 - 0.0223j)x(n-2)$$

$$y(n) = \tilde{y}(n) - 0.055[\tilde{y}(n)]^2 + 0.14[\tilde{y}(n)]^3 \quad (7)$$

A nonlinear active device may be modeled [8] by a complex amplification:

$$G(r) = A(r)e^{j\varphi(r)} \quad (8)$$

where r^2 is the instant power of the input signal and the analytique model of Saleh is used:

$$A(r) = \frac{2}{1 + r^2} \quad (9)$$

$$\phi(r) = \frac{4.0033r^2}{1 + 9.104r^2} \quad (10)$$

IV. NEUREL NETWORKS COMPLEX EQUALIZERS STRUCTURES

The complex equalizers and their training algorithms are extensions of the real ones, obtained by replacing input and output signals, weights, biases, and sometimes activation functions with complex values.

IV. 1 Multilayer Neural Networks

The most used architecture is the multilayer NN equalizer, presented in Fig.3. The output of the communication channel is passed through a LTE followed by a multilayer NN . Usually the backpropagation algorithm is used to determine the LTE and NN coefficients. This algorithm is iterative and minimizes any differentiable cost function,

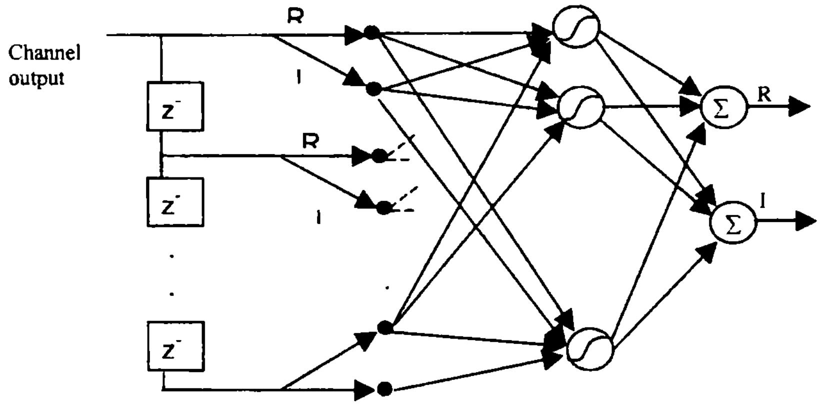

Fig.3 The structure of a multilayer equalizer

such as the mean square error (MSE). In case of complex signals a complex variant of BKP algorithm (CBKP) is needed [9], which has all the drawbacks of the real one. Due to the complexity of the error surface the CBKP algorithm may not converge into the global minimum, but into a local one, that may be an unaccepptable solution. MLP networks trained with BKP need a lot of iterrations to convergence, so a long training time. Another problem that arises in a complex MLP is the selection of the activation function. In case of real signals most activation functions are limited and continuous. In case of complex signals all the normal activation functions may not be limited, except constants.

To satisfy the conflicting relationship between the boundedness and the differentiability of a complex function, there have been developed two CBKP algorithms, one that uses a fully complex activation function (FCBKP) [11] and the other a split complex activation function (SCBKP) [9]. To solve the problem of slow convergence another training algorithm was developed, the complex resilient propagation CRPROP detailed in [9]. CRPROP can provide much faster convergence than CBKP and thus also a smaller computational load. Performance comparisons made in terms of bit error rates (BERs) and computational complexity show that the MLP network trained with complex RPROP algorithm achieves approximately as good bit error rates as the MLP network trained with complex backpropagation, but with clearly smaller computational cost [9].

IV.2 Radial Basis Function Networks

Comparative to MLP, complex RBF networks have a real activation function. They have a simple structure and often provide a faster and more robust solution to the equalization problem. In addition, the RBF neural network has a structure similar to the optimal Bayesian symbol decision equalizer. Note that the Bayesian equalizer does not necessarily yield a good MSE performance but provideS the minimum average BER achievable for symbol decision and indirect-modeling equalizer structures. Therefore, the RBF is an ideal processing structure to implement the optimal Bayesian equalizer. That's why the RBF network is an attractive alternative to the MLP neural networks and it will be detailed in the following.

As depicted in Fig.4, the RBF network has two layers, the hidden layer and the output layer. The hidden layer is composed of an array of computing neurons, each having a parameter c_i, vector called center. Each neuron computes a distance between its center and the network input vector. This distance may be of different types and it is subsequently divided by a parameter ρ_i, called width, which is the spread of the corresponding center. The result is passed through a real, nonlinear activation function. $\phi_i(\bullet,\rho_i)$:

$$\phi_i = [\phi(x-c_i)^H(o-c_i),\rho_i], \quad 1 \le i \le n_h \quad (11)$$

where x is the complex input vector of n_h dimension, c_i is the centers vector of the radial basis functions, which is also a complex vector of n_h dimension, ρ_i is the center spread parameter, n_h is the number of computing nodes. The operator $(\bullet)^H=((\bullet)^T)^*$, where $(\bullet)^T$ is the transposition operator and $(\bullet)^*$ is the complex conjugation operator. The nonlinear output function is usually the Gaussian function:

$$\phi(\chi^2,\rho) = e^{-\frac{\chi^2}{\rho}} \quad (12)$$

Similarity with the Bayesian equalizer impose that the spread parameter $\rho=2.\sigma^2$ where σ^2 is the noise dispersion given by relation:

$$\sigma^2 = E\|x(n)-c_i\|^2 \quad (13)$$

where E is the mean, the second order moment

The output layer of the network consists of eight neurons in case of 4 MAQ (two neurons for each class, one for the real part and the other for the imaginary part of each class) with a linear function :

$$f_{RBF}(x) = \sum_{i=1}^{n_h}\phi_i w_i \quad (14)$$

where w_i are the complex weights. According to the relation (12), f_{RBF} becomes:

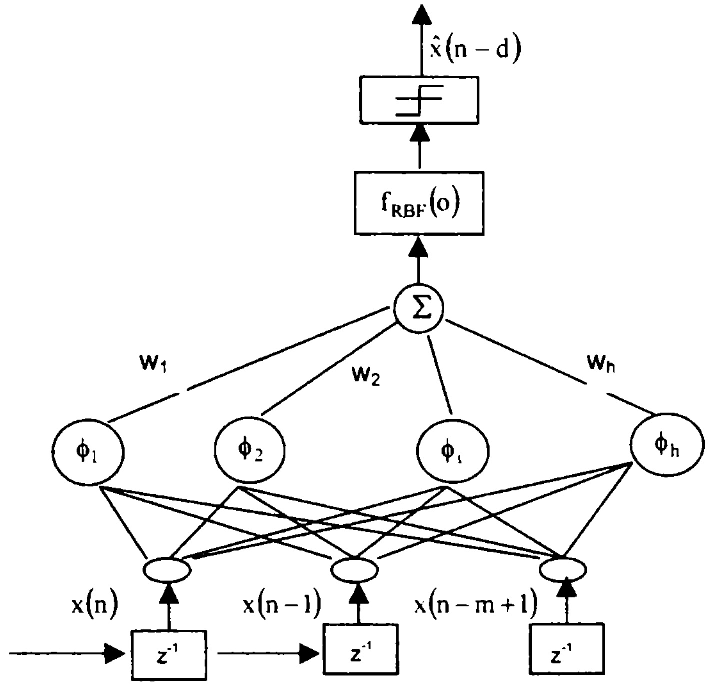

Fig.4 The architecture of a radial basis function equalizer

$$f_{RBF}(x) = \sum_{i=1}^{n_h} w_i . e^{-\frac{(x-c_i)^H (x-c_i)}{\rho_i}} \qquad (15)$$

Several learning algorithms have been proposed to update the RBF parameters. However, the most popular algorithm consists of an unsupervised learning rule for the centers of hidden neurons and a supervised learning rule for the weights of the output neurons. The centers are generally updated using the K-Mean clustering algorithm [8] which calculates the distance between the input vector and the RBF centers vector. The distance may be of different types, but usually the Euclidian norm is used:
The neuron j with a minimum distance is declared winner:

$$j = \operatorname{argmin} \| x(n) - c_i(n) \|, \quad i = \overline{1, n_h} \qquad (16)$$

The winning neuron center is than moved with a fraction η towards the input. The K mean algorithm has some potential problems: classification depend on the initials values of the centers of RBF, on the type of chosen distance, on the number of classes. If a center is inappropriate chosen it may never be updated, so it may never represent a class.

The competitive algorithm penalizing the rival (CAPR) [14] determines not only the winning neuron but also the second winning neuron r. The second winning neuron will move away from the input its center with a ratio γ. All the others neurons will not change their centers vector. So the learning law can be synthesized in the following relation:

$$c_i(n+1) = \begin{cases} c_i(n) + \eta [d(n) - c_i(n)] & if\, i = j \\ c_i(n) + \gamma . [d(n) - c_i(n)] & if\, i = r \\ c_i(n) & if\, i \neq j \text{ and } i \neq r \end{cases} \qquad (17)$$

where η and γ are the learning constants with real values between 0 and 1.

If the learning speed η is chosen much greater than γ, the RBF network will find automatically the number of signal output classes. The algorithm is quite simple and the performances are comparative to all the others reported equalizers [14].

In reference [15] is developed a functional link artificial NN for equalization of QAM signals. FLANN has a single layer structure whose inputs are expanded by using trigonometric polynomials.

In [2] it is proposed a CRBF that uses a stochastic gradient algorithm to adapt all the free parameters of the network simultaneously by using stochastic gradient descent for error criterion. The SG algorithm takes the instantaneous gradient of the squared error and moves the parameters in the opposite direction of their respective gradients.

Reference [12] proposes a sequential learning algorithm referred as complex minimal resource allocation network (CMRAN). Studies using different channels models proved that the equalizer performance is superior to FLNN and SGRBF in terms of BER and computational complexity [12].

High channel orders require correspondingly large equalizer orders which, in turn, lead to a large number of required centers. CRBF network structures become exceedingly complex and impractical as channel orders increase. The center computation techniques proposed in [5] and [13] seek to greatly reduce the number of centers even as equalizer orders are increased, so that good performance is achieved by using higher order equalizers with a small numbers of centers. The reduced network complexity allows its operation to be much faster.

IV.3 Recurrent Neural Networks

Recurrent Neural Networks are the most general case of neural networks since every neuron is connected to every other neuron. In general, RNNs have m external inputs and n fully interconnected neurons. They are highly nonlinear, exhibit a rich and complex dynamical behavior so they are recommended for real time applications. The output of a neuron at the moment n depends not only on the external inputs but also on the previous outputs of the neuron. It is important to note that RNNs with the same structures exhibit different dynamic behavior as a result of using different training algorithms. Consequently, a RNN network is defined only by specifying both its architecture and training algorithm. The most known algorithm for a complex RNN is the complex extension of Real Time Recurrent Learning (CRTRL) algorithm [10]. Because CRTRL is based on the minimization of the MSE by a gradient descent procedure, it is characterized by relatively slow speed of convergence and may suffer from numerical ill conditioning. The computational complexity of RTRL trained RNN equalizers is on the order of n^4 where n is the number of neurons in the network. The small size of RNN equalizers makes them attractive for high speed channel equalization when compared to the complexity associated with other neural equalizer structures.

With the motivation of improving the performance of RNN equalizers, [16] introduced another approach to RNN training. The training algorithm exploits the principle of discriminative learning which minimizes a ··· r ˆu ti nal that is a direc measure of ˙he classification error. The proposed algorithm is a supervised symbol clustering procedure coupled with a statistically robust least square fitting that is generally faster and more stable than the traditional gradient based algorithms. The method was shown to outperform the RTRL algorithm for the equalization of linear and nonlinear channels.

Another alternative to the MSE criterion-based algorithms uses the minimization of the accumulated relative entropy [11]. In this approach, the conditional probability distribution function (PDF) of the transmitted signal is parametrized by a NN. The PDF parameters are estimated by minimization of the accumulated relative entropy cost function. This least relative entropy equalizer has been shown to perform high complex decision boundaries, and to track abrupt changes in a nonlinear channel response whereas the MSE-based multi-layer perceptron equalizer cannot.

In [1] an RBF was used to improve the decision of -- ·-·i ion ˆeedback equalizer DFE. for 16-MAQ signals. passed through a nonlinear device. simulated by relations (8)-(10). The structure of the RNN is presented in Fig.5a. The centers of the RBF network were calculated using the K-Mean algorithm. The LMS algorithm was used in the output layer to label the neurons.

The self-organizing map (SOM) has been combined with a LTE and DFE. The SOM is either in cascade, as in Fig. 5b or in parallel with an conventional equalizer [8]. The Kohonen learning law was used to determine the winning neuron. Each neuron of the SOM was associated with a transmitted symbol through a reference table.

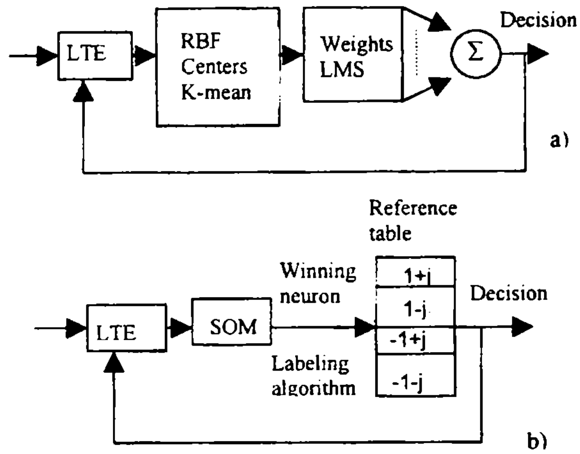

Fig. 5 Structures of recurrent NN
a) LTE-RBF b) LTE-SOM

V SIMULATIONS AND RESULTS

We have tested an RBF equalizer,with 64 hidden neurons, trained with the competitive algorithm penalizing the rival with channel models given by relation (5)-(7), in different conditions of noise. Fig. 7 presents the space state partition of output signals.

Fig.6 The space partition of the 4 MAQ output signals using a RBF equalizer

Fig.8 depicts the MSE evolution during 3000 iterations for different signal to noise ratios and order m of the LTE filter and a delay of d=1. This performance is comparative to other NN equalizers, with a lower computational cost.

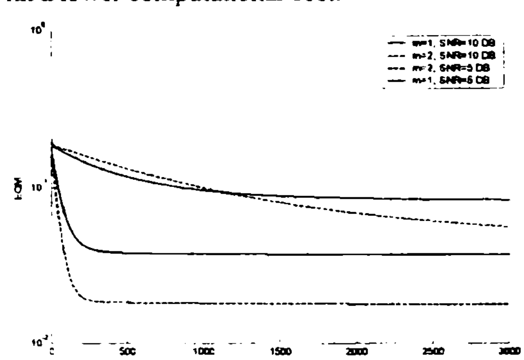

Fig.7 The evolution of MSE

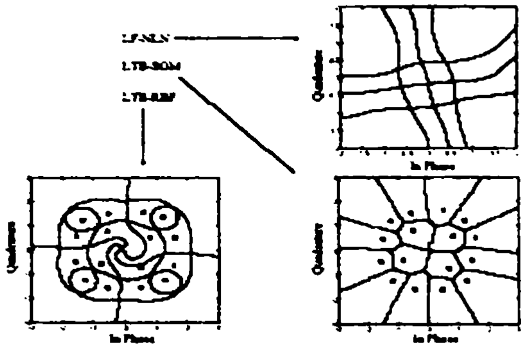

Fig.9 The output space for 16 MAQ signals

In Fig.10 is presented the equalization of 16 QAM signals in case of a satellite mobile channel, simulated with relations (8)-(10), for 150 km/h speed [1]. The LTE- RBF equalizer had the best performance , hundred times better comparative to the LTE error.

VI. CONCLUSIONS

The abundant literature that has grown exponentially in the recent years shows that the communications community gives great interest to NN-based communication systems. This paper gives an overview of NN complex equalizers. It aims at covering the most representative architectures, training algorithms and nonlinear channel models.

The simulations showed that small size RNN based equalizers performed comparably with traditional equalizers for linear channels, but outperformed all other LTE and NN based equalizers, when the channels had severe nonlinear distortions.

In conclusion, RNN are ideal for real time equalization. The best performance was obtained by the LTE-RBF in a feedback structure. It would be interesting to apply in this structure the CAPR training algorithm, which has the advantage of a reduced computational cost and fast convergence.

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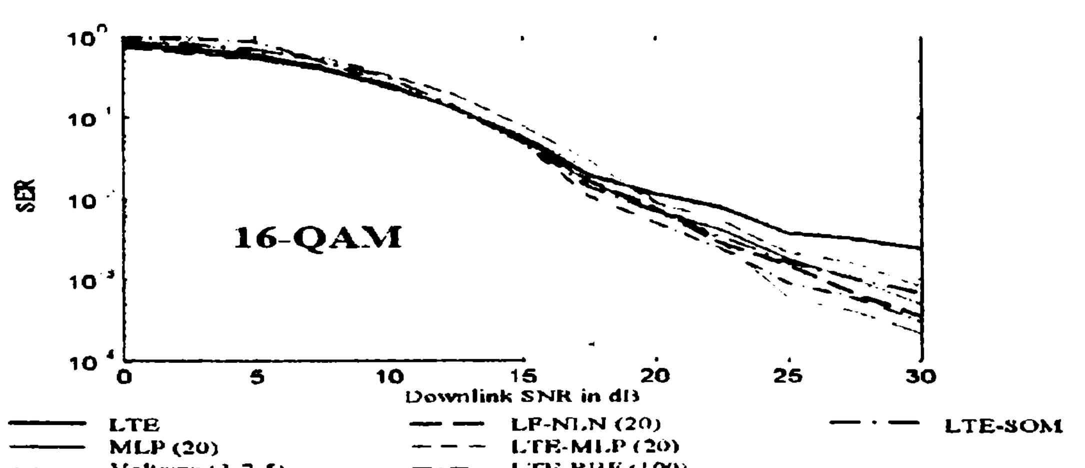

Fiig.10 Signal error rate versus SNR , for different equalizers