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Serial Antiresonance Compensator with Notch Filter

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Abstract – The application of serial antiresonance compensator with notch filter in the speed control system for the cancellation of the undesirable effects of torsion resonance on the motor shaft has been discussed in this paper. Antiresonance compensator is realized as a serial connection of the notch filter and the proportional controller. The notch filter is used to eliminate the resonant picks on the Bode plots of the open loop system response. More efficient time response is attained by adding the proportional controller in the direct loop.

I. INTRODUCTION

Finite stiffness of the applied mechanical elements in electro-motor drives causes torsion resonance which changes the transfer function in the system and makes system stabilization more difficult. There is also explicit tendency to oscillation, which limits servo-system performance. It can lead to undesirable mechanical vibrations, which can damage mechanical elements; it can also cause sustained oscillations (constant amplitude oscillations), having negative effects on system reliability.

When analyzing electro-motor control system, in order to simplify system model, we suppose that mechanical elements have sufficient absolute stiffness, i.e. their stiffness tends towards infinity. In this way, the effects of their compliances on the system performance are neglected. Although such approximation often gives satisfactory results in some analyses, it is only approximate picture of real system condition.

Mechanical elements have finite stiffness, so they are deformed elastically when force is applied. That is why electro-motor shafts are deflected torsionally under the influence of torque. This leads to the angular position difference of some shaft points. Relative angular position depends on angular frequency and system parameters. At some frequencies, angular positions of two different shaft points can be in the opposite phases. This enables mechanical system to accumulate kinetic energy and to increase the value of angular positions. This phenomenon is called torsion resonance, and frequencies at which it appears are called resonant frequencies [1].

Because of undesirable effects of torsion resonance, it is necessary to eliminate it or minimize it at least. There are different methods for minimizing effects of torsion resonance: increasing the shaft diameter, reducing physical separation between inertial elements, minimizing system inertia, applying RC filters. One of the models includes the application of serial resonance compensators with notch filter which add imaginary zeroes in transfer function in order to attract root-loci branches and prevent their crossing into right half of the complex s-plane. The application of notch filters in the cancellation of the undesirable effects of torsion resonance on the motor shaft is discussed in this paper. The notch filter is used to eliminate the resonance peaks on the Bode plots of the open loop system response. More efficient transient response is attained by adding the proportional controller in the direct loop.

The forward-path transfer function of numerous control systems has poles close to the imaginary axis. It is also the case with the servo-drives where there are conditions for torsion resonance. These particular poles can cause close-loop system to be lightly damped, undamped, or unstable. In order to solve the problem, notch filters are used. Notch filter zeroes are selected to be identical with poles which are to be eliminated. In this way, their undesirable effects on close loop system stability are eliminated. The cancellation of poles by means of controller zeroes is the method of notch filter design in s-domain.

Anyway, there are practical problems that limit the application of notch filters. In order to synthesize notch filter, it is necessary to know precisely the positions of resonance system critical poles which are to be compensated. This is almost impossible to achieve for several reasons:

-Because of the almost constant change of working conditions, e.g. changes of inertia as the result of adding and reducing working elements, change of velocity, which causes the change of friction coefficient, possible abrasion of elements as well as their inevitable ageing, etc...... the real poles change their position during the working process.

-Almost all systems in the nature are non-linear. The process of modeling means the simplification and linearization of real complex systems. That is why

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modeled system poles and zeroes do not correspond to real system poles and zeroes.

-The values of notch filter parameters are limited by real available physical elements and cannot be arbitrary.

For all these reasons, precise cancellation of undesirable poles by means of notch filter zeroes cannot be attained practically.

However, it can turn out that precise cancellation of undesirable poles is not necessary [2]. It is enough that these poles and zeroes are sufficiently close. Than, the values of transient response, that is the result of imprecise cancellation, will be practically negligible.

II. DYNAMIC MODEL OF MECHANICAL SUBSYSTEM

Figure 1 shows velocity controlled servo-drive with resonant mechanical subsystem, containing direct current motor and load, both coupled by means of finite stiffness shaft.

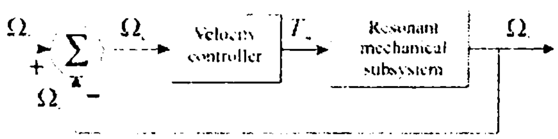

Fig.1 Simplified velocity controlled servo-drive with mechanical subsystem

Mathematical model of the observed mechanical subsystem can be presented in the form of differential equations:

$$T_v(t) = J_L \frac{d\Omega_L}{dt},$$
$$T_v(t) = K(\Theta_m - \Theta_L) + C(\Omega_m - \Omega_L), \qquad (1)$$
$$T_v(t) = J_L \frac{d\Omega_L}{dt}.$$

where $T_m(t)$ is motor torque; $T_v(t)$ is shaft torque; J_m is motor inertia; J_L is load inertia; K is stiffness; C is viscous-friction coefficient of shaft; Θ_m, Ω_m are motor angular displacement and motor angular velocity, respectively; Θ_L, Ω_L are load angular displacement and load angular velocity, respectively.

When Laplace transformation is applied to differential equations (1), implying zero initial conditions, then the transfer function of mechanical subsystem and block diagram presented in Figure 2 are obtained.

$$W_m(s) = \frac{\Omega_L(s)}{T_m(s)} = \frac{Cs + K}{s\left[J_m J_L s^2 + C(J_m + J_L)s + K(J_m + J_L)\right]} \qquad (2)$$

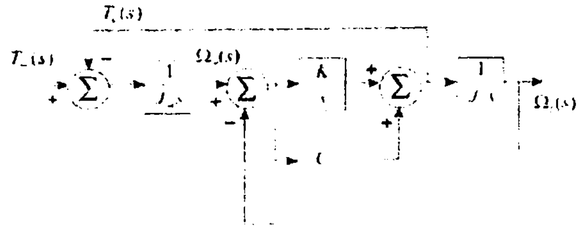

Fig.2 Block diagram of mechanical subsystem with two elastically coupled stiff objects

The data for the particular motor with the load coupled through finite stiffness shaft are shown in Table 1.

Table 1

Symbol	Name	Value
J_m	Motor inertia	$1.83 \cdot 10^{-3}\,\mathrm{kgm}^2$
J_L	Load inertia	$5 \cdot 10^{-3}\,\mathrm{kgm}^2$
K	Stiffness	$10.63\,\mathrm{Nm/rad}$
C	Damping coefficient	$7 \cdot 10^{-3}\,\mathrm{Nm/rad/s}$

Replacing numerous values of direct current motor parameters into equation (2), the mechanical subsystem transfer function can be presented as:

$$W_m(s) = \frac{\Omega_L(s)}{T_m(s)} = \frac{7.6503(s + 1.519 \cdot 10^5)}{s(s^2 + 5.225 \cdot 10^{-2} s + 7935)} \qquad (3)$$

Servo-drive velocity controller, shown in Figure 1, has the form of unit proportional converter of velocity error signal into motor torque, which means that its proportional constant is $K_a = 1\,\mathrm{Nm/rad/s}$.

The forward-path transfer function is the result of multiplication of controller proportional constant and the transfer function of mechanical subsystem

$$W_s(s) = \frac{\Omega_L(s)}{\Omega_e(s)} = K_a W_m(s) = \frac{7.6503(s + 1.519 \cdot 10^5)}{s(s^2 + 5.225 \cdot 10^{-2} s + 7935)} \qquad (4)$$

The Bode plots of the forward-path transfer function $W_s(s)$ are shown in Figure 3.

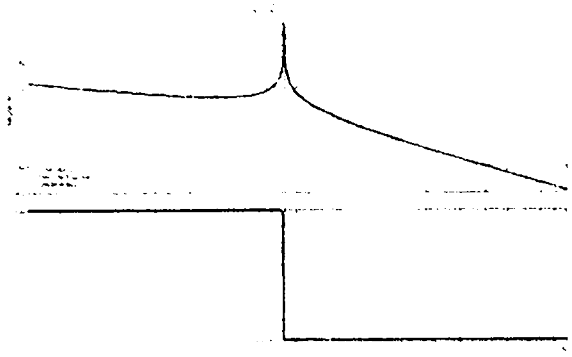

Fig.3 Bode plots of open-loop system

The Bode plots show the resonance peak that is the result of the forward-path transfer function poles, which are close to imaginary axis. These poles are the result of finite stiffness of the shaft that connects motor and load, i.e. the result of shaft torsion resonance; they are also the cause of the observed drive instability. For the observed system these poles are: $-0.0261 \pm j89.1$

III. *NOTCH* FILTER DESIGN

A. *Time-domain design*

Suppose that the desired system performances are specified by the following demands:

- The steady-state error due to unit ramp input, should have not be more than 10 present, $e(\infty) \le 0.1 = 10\%$.
- Maximum overshoot $P\% \le 5\%$.
- Rise time $t_r < 0.1\,\text{s}$.
- Settling time $t_s < 0.2\,\text{s}$.

In order to compensate the effect of torsion resonance, it is necessary to eliminate complex poles $-0.0261 \pm j89.1$ by means of designing serial antiresonance compensator with notch filter, Fig.4. Therefore, the transfer function of notch filter should be as follows

$$W_{notch} = \frac{s^2 + 5.225 \cdot 10^{-2} s + 7935}{s^2 + 2\zeta_p \omega_n s + \omega_n^2}. \quad (5)$$

The forward-path transfer function of compensated system based on (4) and (5) is

$$W(s) = W_s(s) W_{notch}(s) = \frac{7.6503 \cdot (s + 1.519 \cdot 10^5)}{s(s^2 + 2\zeta_p \omega_n s + \omega_n^2)}. \quad (6)$$

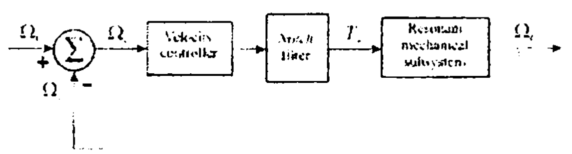

Fig.4 Servo-drive with notch filter including compensation effects of torsion resonance

Based on the need for steady-state error, if there is unit ramp signal input, it is possible to define the limit for notch filter natural undamped frequency ω_n.

Since the system is type 1 [2], the ramp error constant is

$$K_v = \lim_{s \to 0} sW(s) = \frac{7.6503 \cdot 1.519 \cdot 10^5}{\omega_n^2} = \frac{1.162 \cdot 10^6}{\omega_n^2}. \quad (7)$$

For the unit ramp input, the steady-state error of the

$$e(\infty) = \frac{1}{K_v} = \frac{\omega_n^2}{1.162 \cdot 10^6}, \quad (8)$$

and due to condition, $e(\infty) \le 0.1$, it follows

$$\omega_n \le 341. \quad (9)$$

Suppose that $\omega_n = 90$ and $\zeta_p = 0.9$. Notch filter equation becomes

$$W_{notch} = \frac{s^2 + 5.225 \cdot 10^{-2} s + 7935}{s^2 + 162 s + 8100}. \quad (10)$$

The Bode plots of notch filter, which is defined by (10), are shown in Fig.5.

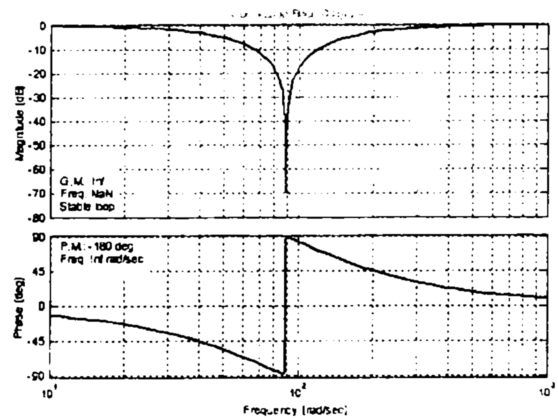

Fig.5 Bode plots of notch filter with transfer function (10)

Based on (4) and (10), the forward-path transfer function of compensated system is

$$W(s) = W_s(s) W_{notch}(s) = \frac{7.6503 \cdot (s + 1.519 \cdot 10^5)}{s(s^2 + 162 s + 8100)}. \quad (11)$$

Servo-drive unit-step response in close-loop system, whose forward-path transfer function is (11), is shown in Fig.6. Response has the following characteristics:

- Maximum overshoot $P\% = 80.4\%$.
- Rise time $t_r = 0.0142\,\text{s}$.
- Settling time $t_s = 1.62\,\text{s}$.

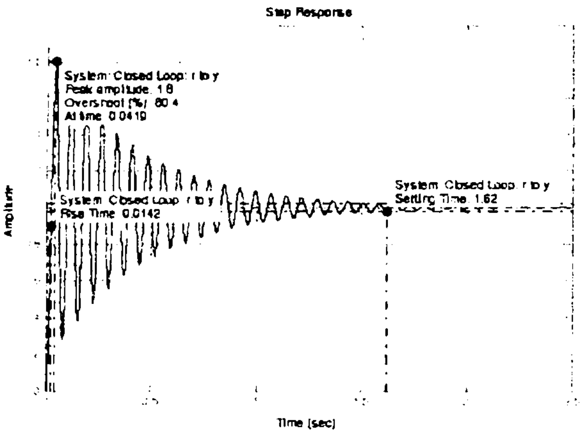

Fig 6 Unit-step response of compensated servo-drive

The system is stable, but the system performances are not achieved. To achieve desired system performance the synthesis of serial proportional controller is one of possible design solutions.
The Bode plots of the forward-path transfer function in Figure 7 shows that the application of notch filter eliminates resonance peak, which causes more efficient system performance in time domain.


Fig. 7 Bode plots of forward-path transfer function of compensated servo-drive

When serial proportional controller with the transfer function $W_c(s) = K_P$ is added, servo-drive can be presented as in Figure 8.

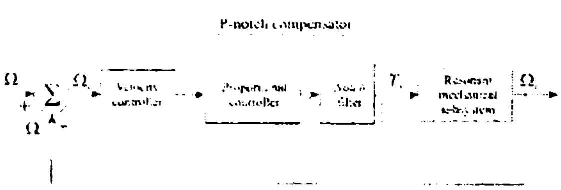
Fig.8 Servo-drive with notch filter and proportional controller

The forward-path transfer function for the system in Fig. 8 is

$$W(s) = W_c(s)W_s(s)W_{notch}(s) = K_P \frac{7.6503 \cdot (s + 1.519 \cdot 10^5)}{s(s^2 + 162s + 8100)} \quad (12)$$

Applying Routh criterion we can define the limitation for the controller proportional constant, K_P, so that the system should be stable,

$$0 < K_P < 1.129 . \quad (13)$$

The demand for steady-state error, if there is unit ramp input, is the additional limitation for proportional constant K_P.
The ramp error constant K_v is

$$K_v = \lim_{s \to 0} sW(s) - K_P \cdot 146.45 . \quad (14)$$

The steady-state error is

$$e(\infty) = \frac{1}{K_v} = \frac{1}{K_P \cdot 146.45} \leq 0.1 , \quad (15)$$

then it follows that

$$K_P \geq 0.0682 . \quad (16)$$

Based on (13) and (16) we obtain that

$$0.0682 \leq K_P \leq 1.129 \quad (17)$$

The root locus of system characteristic equation when $0 \leq K_P < \infty$ is presented in Figure 9. For the values $0 < K_P < 0.111$, dominated pole is real, and as K_P increases, it moves from the origin. This causes more efficient time overdamped response. For the values $0.1 < K_P < 0.111$, all three roots of characteristic equation are real.
For the values $0.111 < K_P < \infty$, dominated pole appears in conjugate-complex pair and it gets closer to imaginary axis by increasing K_P. Because of the decrease of damping factor, overshoot appears and it increases more and more as K_P increases. Although rise time decreases, settling time increases for $K_P > 0.2$, and for $K_P > 1.129$ the system becomes unstable.

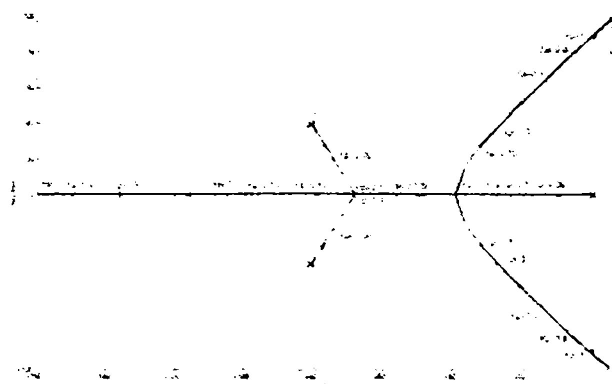
Fig.9 Root locus of servo-drive with P- notch compensator

Table 2 presents the results obtained by synthesis in time domain for the various values of proportional constant K_P. Table 2 shows that the demanded performances are satisfied for the values $0.1 < K_P < 0.17$.

Table 2

K_P	$P\%$	$t_r\,(s)$	$t_s\,(s)$	*Roots of characteristic equation*	
0.08	0	0.145	0.27	-16.2	-72.9±j20.6
0.1	0	0.106	0.195	-24.6	-68.7±j0.118
0.105	0	0.098	0.179	-28.2	-54.8 -79
0.11	0	0.092	0.166	-34.6	-44.5 82.8
0.15	2.02	0.061	0.133	-97	-32.5±j27.2
0.[illegible]6	[illegible]7	0.056	0.[illegible]46	[illegible]	-31.[illegible]j2[illegible].8
0.17	4.44	0.052	0.146	-101	-30.4±j32.1
0.18	5.78	0.049	0.144	-103	-29.5±j34.1
0.2	8.56	0.044	0.137	-106	-27.8±j37.6
0.5	44.5	0.021	0.265	-134	-14.1±j64.4
0.8	68.3	0.016	0.606	-149	-6.31±j78.6
1	80.4	0.014	1.62	-157	-2.31±j85.9

Figure 10 shows the system unit-step responses for various values of controller proportional constant K_P. The increase of controller proportional constant K_P leads to the decrease of rise time, but to the increase of overshoot.

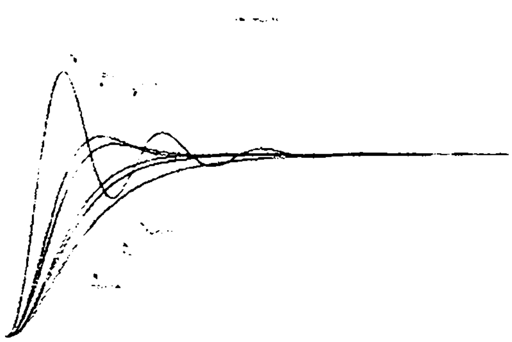

Fig. 10 System unit-step response for the various values of controller proportional constant

B. *Frequency-domain design*

In the Bode plots of uncompensated system the forward-path transfer function in Figure 3, the resonance peak of 68.95 dB exists as the result of torsion resonance and it is formed at the frequency $\omega = 89.1\,\text{rad/s}$. In order to "level" this resonance peak and eliminate undesirable effects of motor shaft torsion, it is necessary to achieve attenuation of approximately 70 dB, which can be seen in Figure 3

$$|W_{notch}(j\omega_n)| = -70\,\text{dB} = \frac{\zeta_z}{\zeta_p} = \frac{0.000293}{\zeta_p}, \quad (18)$$

where ζ_z is obtained from the notch filter equation, (5). Equation (18) results in $\zeta_p = 0.92743$.

Now notch filter equation is:

$$W_{notch}(s) = \frac{s^2 + 0.0522s + 7935}{s^2 + 165s + 7935}. \quad (19)$$

The Bode plots of notch filter designed in frequency domain are shown in Figure 11.

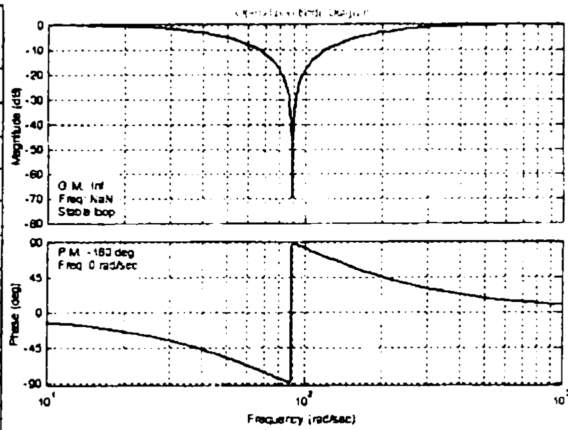

Fig. 11 Bode plot of notch filter with transfer function (19)

Bode plot of the system with notch filter is presented in Figure 12. The observed system with notch filter has phase margin of only $3.79°$, while gain margin is $1.05\,\text{dB}$. Resonance peak is "leveled".

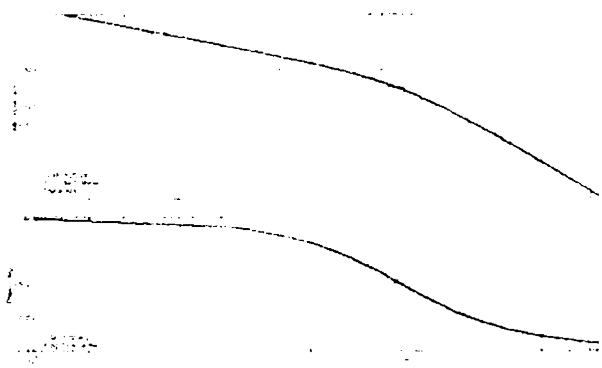

Fig. 12 Bode plots of servo-drive with notch filter

In order to improve system relative stability, proportional controller is designed. The forward-path transfer function is:

$$W(s) = W_c(s)W_s(s)W_{notch}(s) = \\ = K_P \frac{7.6503 \cdot (s + 1.519 \cdot 10^5) \cdot}{s(s^2 + 165s + 7935)} \quad (20)$$

Applying Routh criterion we can define the limitation of controller proportional constant, K_P, in order to achieve system stability.

It can turn out that the allowed range for K_P when notch filter is designed in frequency domain is almost as identical as when notch filter is designed in time domain, because the system characteristic equations are slightly different,

$$0 < K_p < 1.13 .$$
(21)

Suppose that the demanded phase margin is 60° and bandwidth is 20 rad/s. Table 3 presents the results of the synthesis in frequency domain, which are obtained by selecting various values of controller proportional constant.

Table 3

K_P	*Gain margin (dB)*	*Phase margin (°)*	*Resonance peak*	*Bandwidth (rad/s)*
0.08	23	76.3	0	15.74
0.1	21	72.9	0	21.29
0.105	20.6	72.1	0	22.78
0.11	20.2	71.3	0	24.23
0.15	17.5	65.1	0	37.09
0.16	17	63.6	0	39.82
0.17	16.4	62.1	0	42.51
0.18	15.9	60.7	1.01	45.25
0.2	15	57.9	1.04	50.3
0.5	7.07	27.6	2.24	92.15
0.8	2.99	11.1	5.73	113.39
1	1.05	3.79	17.1	124.32

The increase of controller proportional constant K_p leads to "rising" of magnitude curve of the Bode plot while phase curve of the Bode plot remains unchanged, which is shown in Figure 13. All this causes gain crossover frequency to move towards higher frequencies, which leads to the decrease of phase margin as well as to the increase of system bandwidth. Because of the decrease of phase margin while proportional constant K_p increases, relative stability becomes worse; but because of the increase of system bandwidth, the speed of the time response is increased.

The demanded phase margin and bandwidth are obtained for the values $0.1 < K_p < 0.18$.

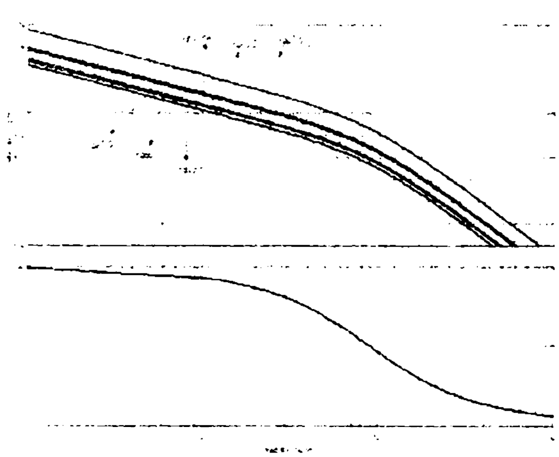

Fig.13 Bode plots of servo-drive with notch-P controller

IV. CONCLUSION

The observed velocity servo-drive with the control plant – resonance mechanical subsystem which contains elastically coupled masses of motor and load – in closed-loop system is unstable.

Notch filter synthesis cancels resonance peak in magnitude curve of the Bode plot, by means of which the major aim – preventing torsion resonance – is achieved. The system becomes stable, but with unsatisfactory performances in time and frequency domain.

One of numerous possible design methods for accomplishing defined design tasks is the synthesis of proportional controller. The allowed range for proportional constant of this controller is defined according to the demands that closed-loop system should be stable as well as to the demanded steady-state error when there is unit-ramp input. The increase of the controller proportional constant causes decrease of the rise time, i.e. the speed increase of the system time response, but also causes increase of the overshoot. Definite acceptable range of controller proportional constant is defined by the other design demands in time domain: desired overshoot, rise time, and settling time.

The effect of controller proportional constant on the performance in frequency domain is such that the increase of proportional constant also causes the increase of bandwidth (in time domain this corresponds to the speed increase of the unit-step response), but phase margin, i.e. relative stability, decreases (which corresponds to the decrease of overshoot in unit-step response). The range of proportional controller constant, which accomplishes demanded system performance in frequency domain, defined by the desired phase margin and bandwidth, corresponds to the range defined in time domain.

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