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On Frequency Measurement by using Zero Crossings

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Abstract – The paper presents the problem of frequency measurement of periodical signals by using the counting of zero crossings of those signals. Theoretical principles are used in order to implement one frequency meter instrument. Because this operation needs digitizing of the measured signal, and it is desired as the proposed instrument to measure a wide range of frequencies with a minimum resolution, o method for aliasing detecting is presented. Thus, if the signal frequency becomes larger than one half the sampling frequency, the user is informed, and the sampling frequency is increased.
Keywords: zero crossings, sampling frequency, aliasing

I. INTRODUCTION

For a periodic waveform, the frequency f and the period T are related by the following operation

$$f=1/T. \qquad (1)$$

Usually, frequency or period measurement of periodic waveform is achieved by means of counter-timer instruments [1], [2], [3] by counting the pulses from a time interval. The hardware of these instruments is not simple. The measurement time can have for instance, values from 1 ms to 10 s [3], depending on desired resolution. One cheap solution is using the data acquisition boards because these devices can implement very many functions, by using of proper software. Thus, one possibility is represented by using counter-timer on-board circuits, but is further necessary a trigger circuit in order to transform the input signal (generally an analog signal) in a square TTL signal. In this paper is proposed a solution which first achieves digitizing of the measured signal, and then by using the obtaining samples, the zero crossings from an observer time are counted. Based on number of zero crossings, the frequency is computed.

The zero crossings are frequently used on frequency measurement [4], [5]. Namely, one half the period of the signal is computed as the number of sampling intervals between two contiguous zero crossings. For this method is necessary a high number of data samples in one signal period in order as the accuracy to be acceptable. This requirement limits the maximum frequency of the measured signal, but has the advantage of a small measurement time.

In the proposed solution, the maximum frequency to be measured is limited only by sampling theorem, i.e. at ½ from sampling frequency. The sampling frequency is automatically selected, in order to obtain the minimum resolution. The measurement time is higher than that from [4] and [5], but its value is not critical because the proposed method is used to an instrument similarly with that from [3]. However, the main idea from this paper is a method for detecting the phenomenon of aliasing. This requirement is necessary because each measurement range has a different sampling frequency, and if the signal frequency becomes higher than one half the sampling frequency, the obtaining result will be false. Thus, if this event happened, the measurement process will be stopped, and then will be resumed with another sampling frequency, which is higher than precedent one. This method can be applied, so will be seen in the next section, only if at the beginning of the measurement process the aliasing is not present. This idea has applications also in other fields where the frequency of acquisition signal is variable and the sampling frequency must be a little bit higher then two-fold this frequency.

The paper is organized as follows. Section 2 presents the theoretical principles that are used in order to implement the presented methods. Section 3 presents the practical achievement of proposed solutions, and section 4 presents experimental results that has been obtained.

II. THE FREQUENCY MEASUREMENT USING THE COUNTING OF ZERO CROSSINGS

Let a signal $s(t)$ as in fig.1, having the frequency $f=1/T$. This signal is sampled by sampling frequency $f_s=1/T_s$, and then is digitized by means of an analog to digital converter.

The period T can be obtained by

$$T=\frac{t_m}{N_p}, \qquad (2)$$

where t_m is the measurement time and N_p is the number of periods. N_p is not mandatory to be an integer. Because

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$$t_m = (N-1)T_s, \quad (3)$$

where N is the number of acquired data samples, and using equations (1) and (2), it follows that the frequency is

$$f = \frac{N_p f_s}{N-1}. \quad (4)$$

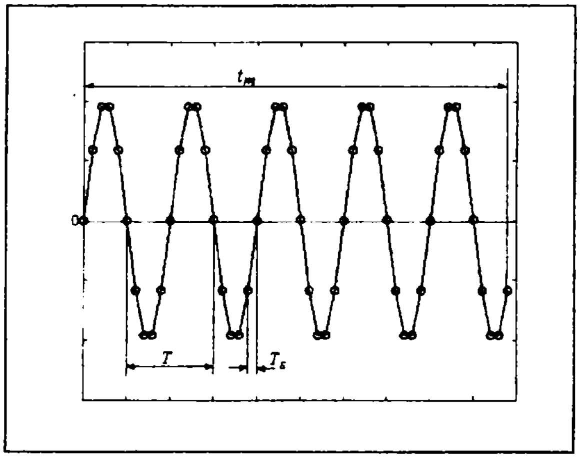

Fig.1 A periodical waveform

The number of periods N_p can be computed based on number of zero crossings N_z, if there are two zero crossings per signal period. Thus, from fig.2 it can be observed that at two zero crossings correspond minimum 1/2 periods and maximum 3/2 periods, or

$$N_p = N_z/2. \quad (5)$$

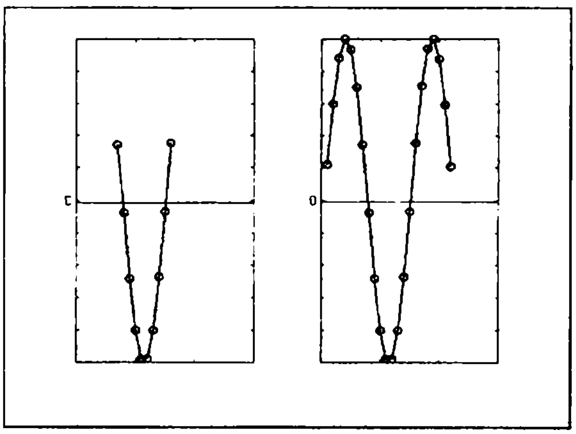

Fig.2 Data samples with two zero crossings

Equation (5) gives the number of periods with a quantization error of ±1/2 periods.

Finally, using (4) and (5) the frequency can be computed by

$$f = \frac{N_z f_s}{2(N-1)}. \quad (6)$$

The relative accuracy in the measurement of frequency f according to (4) is

$$\varepsilon_f = \frac{0.5}{N_p} + \varepsilon_{fs} = \frac{1}{N_z} + \varepsilon_{fs}, \quad (7)$$

where ε_{fs} represents the relative accuracy of sampling frequency and depends on the manner of practical implementation of the sampling process (usually, the on-board counter-timer circuit give the clock for this process). It follows that the single way to minimize ε_f is represented by choosing a higher value for N_p, or equivalently, for N_z. For a given frequency, N_p is direct proportional with N (see equation (4)).

Also, based on (6) it follows that the frequency resolution (also the minimum measurable frequency) is

$$\Delta f = \frac{f_s}{2(N-1)}. \quad (8)$$

If Δf is divided by f given by (6) is obtained the amount $1/N_z$, that is, the first term of ε_f. It follows that, the minimizing of ε_f by N_z is equivalent with the minimizing of frequency resolution.

Thus, it can be say that the two parameters of this method of frequency measurement are f_s and N. The choosing of these parameters must be done based on the next requirements:

1. The maximum value that can be measured is $f_s/2$.
2. The resolution given by (8) must be as minimum as possible.
3. The measurement time given by (3) must be also as minimum as possible.

It is evidently that the requirements 2. and 3. one contradicts each other, because $\Delta f = 1/(2t_m)$. Hence, a compromise must be done in order to choose these parameters. Thus, one possibility is that sampling frequency to be maximum possible, and number of samples variable, in order to modify the resolution, having very high values for low resolutions.. This possibility has the drawback that the processor will be very busy with the reading of data samples, or if the samples are written in a table, this needs a large amount of memory. Therefore, the proposed solution is as the sampling frequency to be as minimum possible function of signal frequency (that is a little amount higher than twofold of signal frequency), and the number of samples to be also variable, but the values will be less than in first possibility.

The using of this method on frequency measurement, needs as previously presented, the verifying of the sampling theorem. If this theorem is satisfied, i.e. $f \le f_s/2$, the number of zero crossings is from (6)

$$N_z = \frac{2(N-1)f}{f_s}, \quad (9)$$

and if this theorem is not satisfied or the aliasing is present, i.e. $f > f_s/2$, the number of zero crossings which are detected is

$$N_{za} = \frac{2(N-1)}{f_s}\left|kf_s - f\right|, \qquad (10)$$

where k is a positive integer, and its value is that the expression $|kf_s - f|$ to be minimum. It can be shown that for $k = 0$, equation (10) is identically with (9).

Thus, for a correct measurement, namely for $f < f_s/2$, when N is constant, the increasing of f is equivalent with the increasing of number of zero crossings, and the decreasing of f is equivalent with the decreasing of number of zero crossings (N_z in equation (9)). If f increases and become higher than $f_s/2$, the number of zero crossings results from equation (10) with $k = 1$ and will decrease. Thus follows the false conclusion that the frequency would decrease. It is specify that in all presented cases, the sampling frequency is constant.

In order to avoid this false measurement, it is desired the detecting of the aliasing phenomenon, that is the case when $f > f_s/2$. One way to do this is by making the data acquisition with two different sampling frequencies.

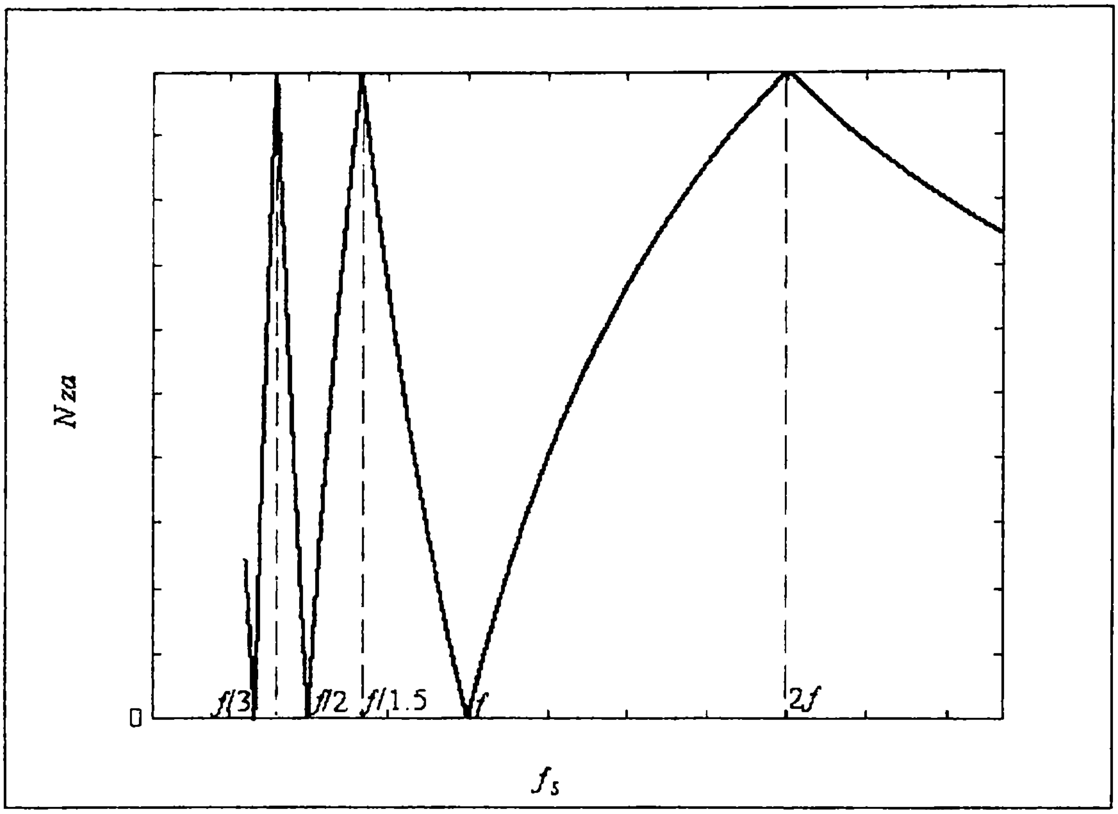

Fig.3 Variation of zero crossings function of sampling frequency

For this purpose, first in fig.3 is presented the variation of number of zero crossings function on sampling frequency for a constant number of samples, if also the signal frequency is constant.. Thus, two statements result from this figure:

1. If sampling theorem is satisfied, $f_s \geq 2f$, the number of zero crossing decreases as sampling frequency increases.

2. If sampling theorem is not satisfied, $f_s < 2f$, the number of zero crossings can increase or can decrease as sampling frequency increases.

From these statements, results that the aliasing phenomenon can be not detected.

However, if sampling theorem is satisfied, and then the signal frequency increases, that $f > f_s/2$ but $f < f_s$, in this case number of zero crossing increases as sampling frequency increases, in opposite with above first statement. In conclusion, the aliasing can be not detected, but the transition from aliasing free to aliasing can be detected.

In order to materialize above statements, one proposes the following idea.

On each measurement, further on current data acquisition with sampling frequency denoted as f_{s1} and corresponding number of zero crossings N_{z1}, it makes an additional data acquisition with $f_{s2} = kf_{s1}$, where k is higher than 1, followed by the computing of the new value of the number of zero crossings, denoted as N_{z2}. In fig.4 are presented the variations of number of zero crossings function on signal frequency, for both values of sampling frequency, f_{s1} and f_{s2}. Also, the number of samples N is constant. From fig.4 it can be seen that, if $f < f_{s1}/2$, then $N_{z2} < N_{z1}$. This is the normal case. If $f > f_{s1}/2$, i.e. the aliasing occurs, the inequality $N_{z2} < N_{z1}$ is satisfied, but only for a little interval, namely for $f \in (f_{s1}/2, f_c)$. $f = f_c$ (denoted as critical frequency) represents the point for which the two curves one intersects. If $f > f_c$, one obtains that $N_{z2} > N_{z1}$, this being the element which signals that aliasing was occured.

It is noted however that this idea for aliasing detecting can be applied only if the measurement process begin in a case for which the aliasing not occur and the increasing of frequency to be that its value to be not greater than sampling frequency.

In order to computes the frequency f_c beginning the aliasing is detected, the expressions of both zero crossings, for $f > f_s/2$ must be equals, as next is presented.

$$\frac{2(N-1)(f_{s1}-f_c)}{f_{s1}} = \frac{2(N-1)f_c}{f_{s2}} \qquad (11)$$

It results that

$$f_c = \frac{f_{s1} f_{s2}}{f_{s1}+f_{s2}} \qquad (12)$$

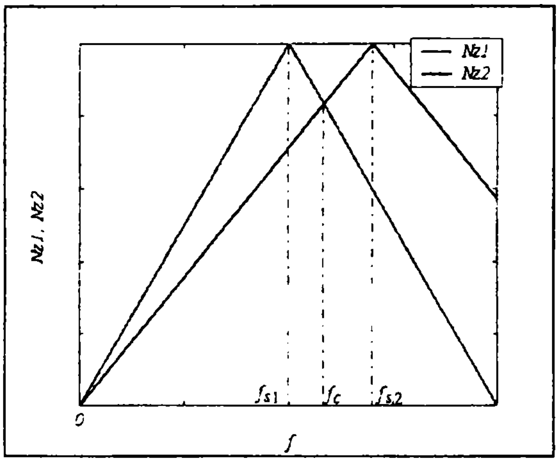

Fig.4 Variation of zero crossings function on signal frequency for two sampling frequencies

In the interval for which $f \in (f_{s1}/2, f_c)$, the aliasing phenomenon can be not detected and therefore this interval must be as little possible.

In order to achieve this requirements, it is necessary as f_{s2} to be as near as f_{s1}, or the coefficient k to be a little amount higher than 1. There are however two restrictions regarding on choosing of f_{s2}.

1. The value must be sufficient high in order to allow the variation of number of zero crossings with least 1 unit.

2. The desired increasing of f_{s2} comparative with f_{s1} must be possible, accordingly of features of data acquisition board (the minimum possible variation of sampling frequency).

III. THE IMPLEMENTED ALGORITHM

Ideas that have been presented in previous section allow the achievement of an algorithm that has the goal on frequency measurement with the minimum resolution.

Thus, the proposed methods were implemented on a data acquisition board of type National Instruments PCI 6023.

Mainly, this algorithm does the following:

-the acquisition of data samples begin at maximum value of sampling frequency, in accordance with features of the used data acquisition board; if it is necessary, the sampling frequency is decreased until a approximate value of frequency can be computed;

- based on previously computed frequency, the optimum sampling frequency and the optimum number of samples are choose;

-the permanent frequency measurement is achieved: the frequency is computed and displayed, and it is checked if the aliasing occurs; if it there are, an message is displayed and the sampling frequency is increased to the next value.

In order to achieve the measurement ranges, it is necessary the settling of the sampling frequency (i.e. the highest limit of range is one half the sampling frequency). Further, the number of samples must also be settled because, so was pointed, these two amounts together establish the frequency resolution. Next, in the table 1, the sampling frequency, the resolution, the number of samples and the measurement time for each range are presented. The values correspond to equations (8) and (3).

Table 1

Range	f_s Hz	Resolution	N	t_m
0-250 Hz	500 Hz	0.1 Hz	2501	5 s
0-1000 Hz	2000 Hz	0.1 Hz	10001	5 s
0-2500 Hz	5000 Hz	1 Hz	2501	0.5 s
0-10 kHz	20000 Hz	1 Hz	10001	0.5 s
0-25 kHz	50000 Hz	1 Hz	25001	0.5 s
0-50 kHz	100000 Hz	10 Hz	5001	0.05 s
0-100 kHz	200000 Hz	10 Hz	10001	0.05 s
0-250 kHz	500000 Hz	10 Hz	25001	0.05 s

It can be seen that for first two ranges, the measurement time has high values. If is not necessary a resolution of 0.1 Hz, the measurement time can be decreased. It is pointed out that in order to check if there is aliasing, it is necessary one more time interval, which is less than the measurement time, as how will be seen. In this time interval the signal frequency must keep constant.

The implemented algorithm contains the next steps:

1. Acquire N_1 = 501 data samples with maximum sampling possible, $f_s = f_{smax}$ (f_{smax}=500000 Hz).

2. Compute the number of zero crossings N_z. If $N_z > N_t$ go to 4. N_t is a thereshold value that allows

computing the frequency with an acceptable accuracy (N_p=50).

3. Acquire N_1 data samples with $f_s/2$. Go to 2.

4. Compute the frequency f using (6). Set $f_s = 2f$.

5. Choose the sampling frequency function of f_s: the less value from table 1 which is higher than f_s. Denote this value by f_{s1}. Also, function on selected sampling frequency, from table 1 results the number of samples N.

6. It is make the current frequency measurement, with the sampling frequency to f_{s1}.

The next two steps are executed continuously.

6.1. Compute N_{z1} at f_{s1} for N samples. Compute frequency f using (6) and display its value.

6.2. Compute N_{z2} at $f_{s2}=kf_{s1}$ for $N/10$ samples. If $N_{z2}>N_{z1}/10$ the aliasing occurs; display the message "Aliasing" and utter a sound; set f_{s1} to the next value accordingly tab.1.

In first three steps, the frequency is computed with an accuracy of 2 percent, because the thereshold N_t has been imposed as 50. The value of 501 for number of data samples allows as the sampling frequency to be decreased to 1/20 from signal frequency.

In order as the needed time for aliasing detecting to be as low as possible, in the second data acquisition only $N/10$ samples has been acquired.

If is necessary, for instance when the measurement signal is changed, the step.6 can be executed only once, and then the algorithm can be resumed beginning with step.1, but in this mode the needed time is higher than that for executing of step.6.

Based on previous presented two restrictions, the constant k has value of 1.05.

Also, next is presented the way for determination of number of zero crossings, where $e(i)$ represents the sample at moment i and N_z represent the number of zero crossings:

For $i=2,\ldots,N$

If $e(i-1)e(i)<0$ or $e(i-1)=0$, $N_z=N_z+1$.

It is specify that before of determination the number of zero crossings, the dc component of signal is removed by using the adaptive LMS algorithm [7].

This algorithm was implemented by a program in C language. For this purpose, the low level functions from NI-DAQ software was used in BorlandC 5.0.

The algorithm is an off-line real time algorithm (see [6]), because it processes a group of N data samples, that previous are stored in the memory. This way was imposed because the functions from NI-DAQ give groups with a pre-established number of data samples, which can be processed only when the acquisition is ready. For this reason, the measurement time is a little amount higher then that from table 1, namely with the required time for computing the number of zero crossings and frequency. However this increasing is visible only for last three ranges, thus the new value of the measurement time for these ranges is about of 0.07s.

IV. EXPERIMENTAL RESULTS

The presented algorithm was experimental tested in order to achieve the desired frequency meter, and the obtained results will be presented in this section.

Thus, a HM 8130 function generator was used in order to apply the signal to be measured. The type of waveform was mainly sine, but also square, triangle or sawtooth were used. This generator displays the value of generated frequency and will be used as reference for comparison with the implemented instrument.

Two categories of experiments have been achieved. First, the accuracy of measurement was verified, for all ranges of the instrument. Second, the manner of detecting the aliasing was tested.

Thus, in order to verify the precision of the instrument, different frequencies were settled to the HM 8130 function generator. For each frequency, a number of 50 measurements were made with the implemented instrument and the obtained results were stored.

In table 2, for each frequency from HM 8130 are presented the distribution of the obtained values f_m (the value and the number of achievements).

Table 2

f (from HM8130)	f_m	
239.30 Hz	239.3 Hz	239.4 Hz
	49	1
729.00 Hz	729.0 Hz	729.1 Hz
	43	7
2207.0 Hz	2207 Hz	2208 Hz
	48	2
8554.0 Hz	8554 Hz	8555 Hz
	45	5
23930 Hz	23930 Hz	23931 Hz
	38	12
41040 Hz	41040 Hz	41050 Hz
	47	3
91670 Hz	91670 Hz	91680 Hz
	40	10
239.28 kHz	239.28 kHz	239.29 kHz
	36	14

From the results from table 2 it can be seen that the quantization error from numbering of periods brings about the frequency resolution. For all frequency, the number of achievements of the results without error is higher than that of the results affected by quantization error.

In order to verify the detection of aliasing, on each measurement range, the frequency of measured signal was modified beginning from values a little amount less than one half the sampling frequency, to values a little amount higher than one half the sampling frequency, and then the value for which the instrument signals aliasing was kept in mind. This value has been compared with the theoretical value, which is obtained using equation (12).

Thus, in table 3 these data that allow the detection of aliasing are presented. f_{cexp} represents the experimental value which has been obtained for signaling of aliasing.

Table 3

f_{s1}	f_c	f_{cexp}
500 Hz	256.09 Hz	256.3 Hz
2000 Hz	1024.39 Hz	1025.5 Hz
5000 Hz	2560.97 Hz	2564.3 Hz
20000 Hz	10243.9 Hz	10210 Hz
50000 Hz	25609.7 Hz	25643 Hz
100000 Hz	51219.5 Hz	51.405 kHz
200000 Hz	102439 Hz	102.05 kHz
500000 Hz	256097 Hz	257.15 kHz

Based on data from table 3 it can be seen that the theoretical idea for aliasing detecting was acceptable verified by experiments.

V. REMARKS

The paper presents an instrument for frequency measurement, which is achieved with a data acquisition board, based on zero crossings counting. The theoretical contributions regarding to aliasing detecting are presented. An algorithm that allows the measurement of frequency and switch among ranges automatically when the aliasing is detected, is also presented. Finally, the experimental results were found in good agreement with the theoretical considerations.

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