Buletinul Ştiinţific al Universităţii "Politehnica" din Timişoara

Seria ELECTRONICĂ şi TELECOMUNICAŢII
TRANSACTIONS on ELECTRONICS and COMMUNICATIONS

Tom 49(63), Fascicola 2, 2004

BER Performances of a Differential OFDM System in Fading Channels

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Abstract – In an OFDM system, various modulation methods can be used in order to encode the binary information. If a differential phase modulation scheme is chosen, data can be encoded in the relative phase of consecutive symbols in each subchannel or in the relative phase of symbols in the adjacent subchannels. The two methods exhibit two essentially different behaviors in fading conditions.

In this paper, we shall investigate the BER performances of both modulation types. The performance evaluation is based on computer simulation. We will consider a multipath fading channel, as met in mobile communication systems.

Keywords: OFDM, differential, fading

I. INTRODUCTION

OFDM (Orthogonal Frequency Division Multiplexing) is one of the most promising modulation techniques that were proposed for being used in the 4th generation wireless systems. In a typical mobile radio channel the transmitted signal is subjected to multipath fading which generally exhibits time selectivity (commonly referred to as Doppler effect) and frequency selectivity [1], [2]. The influence of the ISI (Inter-Symbol Interference) can be reduced by increasing the duration of the transmitted symbol. Using OFDM, the high-rate data sequence to be transmitted is split into a large number of lower speed symbol streams, each of them modulating a different carrier. The carrier spacing is selected such that all carriers used are orthogonal each other over a symbol interval. As it is well known, the orthogonal signals can be separated at receiver by correlation techniques. In addition, a cyclic prefix (a copy of the last several samples of an OFDM "frame") is inserted at the beginning of each OFDM symbol, in order to counteract the inherent time-dispersive nature of the channel, preventing two or more symbols to interfere each other [3], thus inducing ISI. The cyclic prefix gives also an "appearance" of periodicity or circularity to the signal, facilitating the equalization process to the receiver.

The lengthening of the symbol duration, introduced in order to combat the frequency-selectivity is however limited by the time-variant nature of the channel that generates the Doppler effect. Larger the symbol duration, higher the probability that the channel parameters vary during the transmission of an OFDM frame giving rise to frequency offsets of the carriers, thus destroying their orthogonality and generating inter-carrier interference (ICI).

The transmitter and receiver for OFDM can be efficiently implemented using Fast Fourier Transform (FFT), a rapid mathematical algorithm of processing Discrete Fourier Transform (DFT).

The data symbols that modulate multiple orthogonal carriers in OFDM are obtained using a classical digital modulation scheme. Various modulation methods could be employed such as BPSK, QPSK (also with their differential form) and QAM with several different signal constellations.

If a differential phase modulation is chosen (the particular case of an OFDM-DBPSK system) there are two options to perform it. Thus, data can be encoded in the relative phase of consecutive symbols in each subchannel (corresponding samples in adjacent OFDM symbols, or frames), obtaining an inter-frame differential modulation. On the other hand, data can be encoded in the relative phase of symbols in adjacent subchannels (consecutive samples of an OFDM symbol), achieving an in-frame differential modulation. The two methods exhibit two essential different behaviors in fading conditions.

In this paper we realize a performance comparison of the two methods, rarely reported in the literature, focusing on the differences between them. The performance evaluation of both methods is based on computer simulation. We will consider a multipath fading channel, as met in mobile communications systems.

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II. SYSTEM DESCRIPTION AND FADING CHANNEL MODEL

In OFDM, the available bandwidth is partitioned into N subchannels. The desired high-rate symbol stream is achieved by simultaneously transmitting N slower rate substreams using N orthogonal subcarriers. The binary data to be transmitted is differentially encoded using a DBPSK modulation scheme, obtaining a sequence of complex data symbols (fig. 1).

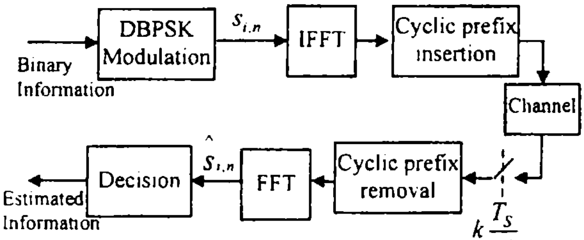

Fig 1: The OFDM transceiver model

There are two possibilities to perform differential modulation in the presented OFDM scheme. Data can be encoded in the relative phase of adjacent symbols in each subchannel (correspondent samples in two consecutive OFDM symbols) or in the relative phase of samples transmitted in adjacent subchannels, that is consecutive samples of an OFDM symbol (see fig. 2). Since the IFFT block accepts N parallel samples to its entry, the whole difference of the two methods can be thought as follows: if the phase modulation is separately achieved on each of the N parallel streams that constitute the entry to the IFFT block, then we are in the case of the first presented modulation type, namely an inter-frame modulation is performed (see fig. 2a). If the modulation is made on the serial stream, prior to the parallel conversion required by IFFT, then an in-frame modulation is chosen, since N consecutive serial samples will simultaneously modulate N orthogonal carriers, forming an OFDM symbol (fig 2b).

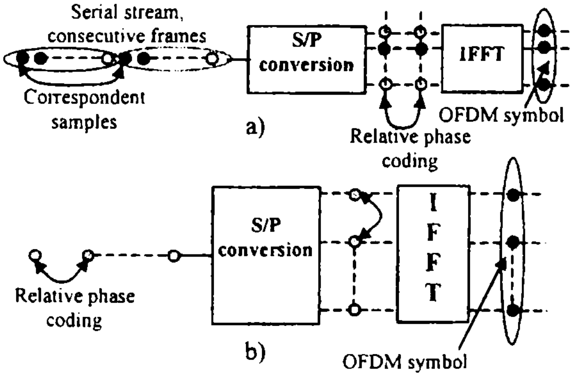

Fig.2: (a) Inter-frame modulation (b) In-frame modulation

Both methods have an irreducible error rate because of the random change of the relative phase, caused by the fading channel. In the first method the distortion caused by the multipath fading is indicated by the Doppler spectrum. In the second case, the channel multipath intensity profile and the length of the OFDM symbol indicate the phase change rate due to fading conditions. Therefore, the two methods exhibit essentially different behaviors although both encode the data differentially. Another major difference between the two methods (that gives them their specific name) is that the consecutive OFDM symbols are interconnected through differential encoding in the first case (inter-frame differential modulation), while no successive OFDM symbols connection is realized by the second method (in-frame differential modulation), where the differential encoding is performed on the samples belonging to the same frame (or to the same future OFDM symbol), as illustrated in the fig. 2b.

After the differential encoding of the binary message using one of the two methods presented above, the sequence $s_{i,n}$ is obtained, $s_{i,n}$ denoting the n-th symbol of the i-th frame, where $(0 \le n \le N-1, -\infty < i < \infty)$. The n-th carrier is modulated by the samples $\{s_{i,n}, -\infty < i < \infty\}$ and the modulated carriers (orthogonal one-another) are added together to form the OFDM symbol to be transmitted. In a practical implementation, the N samples of the OFDM symbol corresponding to the i-th frame are generated by processing $\{s_{i,n}\}$ stream using the fast implementation of the Inverse Discrete Fourier Transform (IDFT) (see fig. 1). In order to combat the inter-symbol and inter-carrier interference introduced by the frequency selectivity and the time selectivity of the radio channel, each OFDM symbol is preceded by a cyclic prefix of L samples. The cyclic prefix is a circular extension of the time domain samples, being obtained by copying the last L samples of the OFDM symbol in the front of it. The i-th transmitted symbol (including the prefix) contains N+L time domain samples, of which the m-th sample is given by the equation below:

$$g_i(m) = \sqrt{\frac{E_S}{N+L}}\sum_{n=0}^{N-1} s_{i,n} e^{j2\pi\frac{nm}{N}}, \quad m = -L, \ldots, N-1 \qquad (1)$$

Assuming the data symbols are statistically independent and having a unit average energy, the transmitted average energy per symbol equals E_S. The transmitted signals can be expressed in complex form as:

$$s(t) = \sum_{i=-\infty}^{\infty} p(t - iT_S) g_i(t) \qquad (2)$$

where $g_i(t)$ represents the analogical waveform corresponding to the OFDM symbol, obtained after a DAC conversion of the sequence $\{g_i(m)\}$, m=0,1,...,N-1. p(t) is the pulse-shaping waveform of each symbol, defined as:

$$p(t)=\begin{cases}1, & for\ -\Delta\le t\le t_s\\ 0, & otherwise\end{cases} \qquad (3)$$

$T_S=\Delta+t_s$ stands for the total duration of an OFDM symbol, composed by the cyclic prefix period (Δ) and by the observation period (t_s). The fading channel (assuming Rice conditions) can be modeled as a 3-ray tapped delay line with one line-of-sight (LOS) path and two multipath components. If $h(t,\tau)$ denotes the channel impulse response at time $t-\tau$, it can be expressed as:

$$h(t,\tau)=\sqrt{2P_S}\,\delta(\tau)+\sqrt{P_1}\,a_1(t)\delta(\tau-\tau_1)+\sqrt{P_2}\,a_2(t)\delta(\tau-\tau_2) \qquad (4)$$

where P_S is the power of LOS signals, P_1 and P_2 are the powers of multipath replicas. The channel impulse response above can be viewed as a summation of the LOS deterministic signal and two attenuated ($a_1(t)$ and $a_2(t)$ are independent time-varying complex Gaussian random processes with maximum Doppler shift f_d, accounting for these attenuations) and delayed replicas (τ_1 and τ_2 represent the delays of the two multipath components). An important parameter characterizing the Rician fading channel is the Rice factor, defined as the ratio of the deterministic LOS component power P_s and the multipath components power $P_m=P_1+P_2$, i.e. $K=P_s/P_m$. As a special case, the channel is AWGN when $K\to\infty$, while Rayleigh fading conditions are met for $K=0$.
The received signal can be written as [4]:

$$r(t)=\int_0^{\infty}s(t-\tau)h(t,\tau)d\tau+n(t) \qquad (5)$$

where n(t) is a complex Gaussian noise and $h(t,\tau)$ is the impulse response of the multipath fading channel at the time $t-\tau$. At the receiver, after doing FFT to the signal, the output of each m-th subchannel can be obtained as:

$$r_{m,i}=\frac{1}{t_S}\int_{iT_s}^{t_s+iT_s}\left\{\int_0^{\infty}s(t-\tau)h(t,\tau)d\tau+n(t)\right\}\times e^{-j2\pi f_D(t-iT_s)}dt \qquad (6)$$

Finally, the differential detector decides what symbol was transmitted.

III. SIMULATION RESULTS AND DISCUSSIONS

The BER performance of an OFDM system with both DBPSK in-frame and inter-frame modulation was studied by the means of computer simulation. To simplify the computer implementation, the cyclic prefix duration is considered to be equal to the serial symbol duration, i.e. $T=\Delta$ for all the simulations. Two-ray Rayleigh fading conditions, with equal power of the two multipath components are considered for channel simulation. The BER computation was averaged over 20000 transmitted OFDM symbols. Neither channel coding nor further equalization to the receiver were considered at this stage. A comparison of the two methods is made, studying the influence of the block length N, of the channel multipath delay spread and of the Doppler shift introduced by the time-variant character of the channel on the BER performance in both in-frame and inter-frame DBPSK-OFDM system. We emphasize the essential different behaviors of the two methods with respect to the parameters presented above.

The BER performances of the DBPSK-OFDM system in a Rayleigh fading channel, as a function of the normalized delay of the second multipath τ_2/T is illustrated in the figures 3,4.

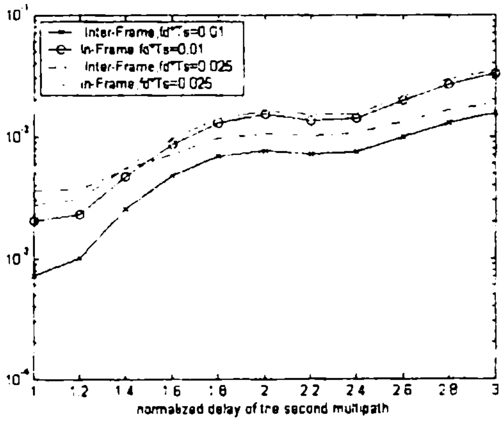

Fig. 3: BER performances of inter-frame and in-frame modulation in a Rayleigh fading channel, SNR=40dB, P_1/P_2=0dB, N=32

In the figure 3, one can observe that the inter-frame modulation is significantly more sensitive to the Doppler shift than the in-frame modulation. Thus, at the two normalized Doppler shifts (f_D*T_S) taken into account, the performance of the in-frame DBPSK system is almost identical, especially for an important multipath delay of the channel. On the other hand, inter-frame modulation performs significantly better for low values of the maximum Doppler shift, proving

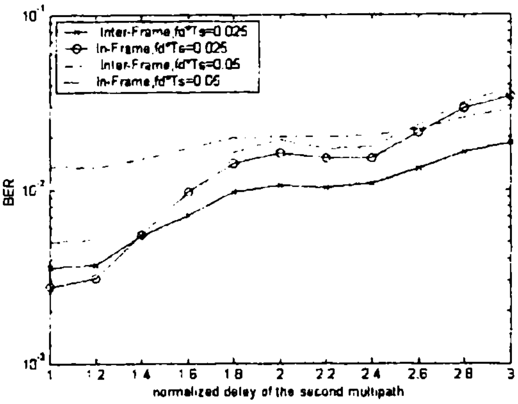

Fig. 4: BER performances of inter-frame and in-frame modulation in a Rayleigh fading channel, SNR=40dB, P_1/P_2=0dB, N=32

a sensitivity of this method to the time-variant channel character. The same observation becomes more obvious regarding the figure 4, where another difference between the two Doppler shifts is taken into account.

The BER performance of the inter-frame DBPSK-OFDM system for three different Doppler shifts is plotted in the figure 5, in order to stress the effectiveness of this parameter. It is shown that the maximum Doppler shift has a significant influence on the BER, especially when the delay of the second multipath is small. For large delays of the second multipath the main amount of errors is brought by the ISI introduced by the multipath components, which confirms the conclusion in [5], respectively in [4] for an in-frame DBPSK modulation.

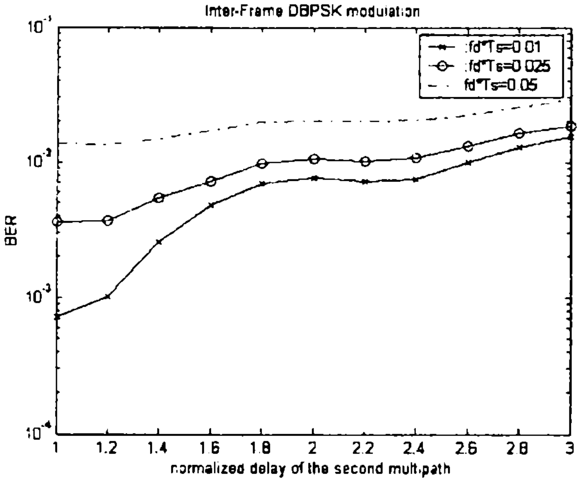

Fig. 5: BER performances of inter-frame DBPSK modulation in a Rayleigh fading channel, SNR=40dB, P_1/P_2=0dB, N=32

In the figure 6, we plotted the BER performance of both inter-frame and in-frame OFDM-DBPSK systems with respect to the average SNR per bit and the parameter used is still the normalized maximum Doppler shift (f_D*T_S).

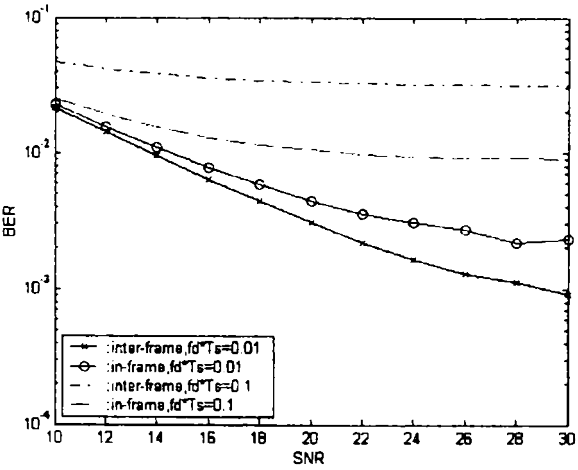

Fig. 6: BER performances of inter-frame and in-frame DBPSK modulation in a Rayleigh fading channel, $\tau_2/T=1$, P_1/P_2=0dB, N=32

The two inner curves correspond to in-frame modulation, showing the little sensitivity of the method to the variation of the maximum Doppler shift parameter comparing to the inter-frame modulation (whose performance is indicated by the two outer curves). If at low maximum Doppler shift inter-frame modulation performs better, once the value of this parameter grows, the in-frame modulation method becomes more efficient. It can also be observed that at significant Doppler shifts, the two methods exhibits a very poor improvement of BER performance, with respect to SNR, especially for the inter-frame modulation.

The influence of the block length N on the BER performance in both modulation types is illustrated in the figure 7, where maximum Doppler shift is considered to be constant. As stressed in [6], the in-frame OFDM-DBPSK system significantly improves its performance when the block (or, equivalently, the OFDM symbol) length increases, considering the same multipath delay spread of the channel impulse response. On the other hand, it turns out that the BER performance of inter-frame modulation is almost identical for the different values chosen for the parameter N. In the three simulated situations (N=16,32,64), the system performs to within 1-2dB spread of the results. It can be asserted that the performance obtained using this modulation type is very little sensitive to the OFDM symbol duration.

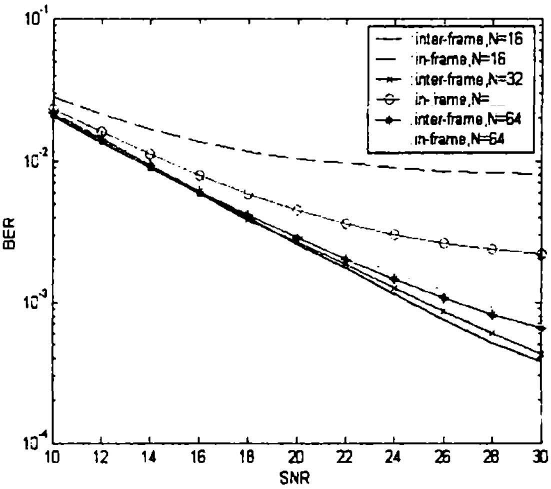

Fig. 7: BER performances of inter-frame and in-frame DBPSK modulation in a Rayleigh fading channel, $\tau_2/T=1$, P_1/P_2=0dB, f_D=0.0001

Figure 8 shows the BER performance evaluated over a range of normalized delay spreads of the channel impulse response. Unlike the previous evaluation, inter-frame modulation improves its performance significantly when the OFDM symbol length N increases. The same expected pattern is exhibited by the in-frame modulation method. A comparative analysis illustrates that in-frame modulation is slightly more sensitive to the parameter N than the inter-frame modulation, when the performance is evaluated against the normalized delay of the second multipath. At small values for the delay of the second reflected path (comparable with the serial symbol duration) no correlation between the frame length and the BER performance can be asserted, since the system performs better for N=16 than for N=64.

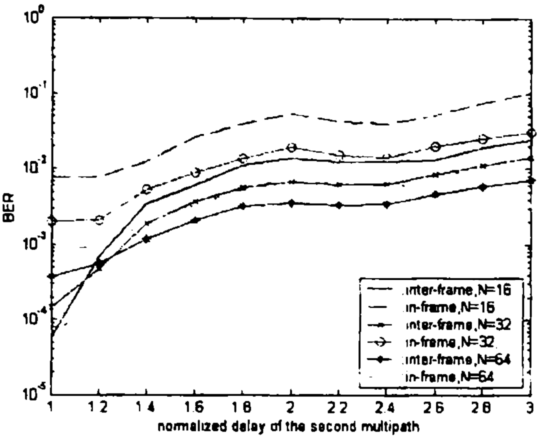

Fig. 8: BER performances of inter-frame and in-frame DBPSK modulation in a Rayleigh fading channel, SNR=40dB, P_1/P_2=0dB, f_D=0.0001

In the figure 9, the effect of multipath delay spread on the both methods is studied. The maximum Doppler shift was kept constant while the BER performance against signal-to-noise (SNR) ratio was plotted for two different values of the normalized delay of the second multipath component. Considering a small value of the mentioned parameter, the in-frame OFDM-DBPSK system performs to about 18dB better than the inter-frame system. To lie in such a situation, a correspondent value was chosen for the normalized Doppler shift (f_D*T_S=0.025). When a three times bigger value was considered for the normalized delay of the second multipath component inter-frame modulation performed better, despite the significant value of the Doppler shift. One can conclude that in-frame modulation is more sensitive to the multipath delay spread introduced by the inherent dispersive nature of the radio channel.

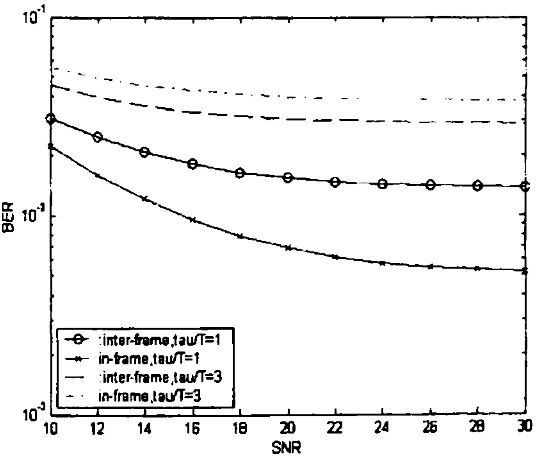

Fig. 9: BER performances of inter-frame and in-frame DBPSK modulation in a Rayleigh fading channel, P_1/P_2=0dB, f_D*T_S=0.025. N=32

IV. CONCLUSIONS

In this paper we have studied the BER performance of an OFDM-DBPSK system with two distinct phase modulation types. The principles of both in-frame and inter-frame modulation in an OFDM transmission scheme were briefly exposed, accentuating on their differences. The essential different behavior in multipath fading conditions was emphasized by means of computer simulation. The inter-frame modulation system, while generally performing better has though shown to be more sensitive at the variation of the Doppler shift parameter. The in-frame modulation method allows significant performance improvement by increasing the data-block length. The multipath delay spread degrades the BER performance of both studied modulation types. Even if at delay spreads that significantly exceed the cyclic prefix duration the performance is only slightly improved increasing the signal-to-noise ratio, the inter-frame modulation proved to be more resistant to the inter-symbol interference introduced by the multipath delayed components.

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