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A Low Complexity Decision Feedback Equalization for Sparse Wireless Channels

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Abstract—In this paper, a low complexity decision feedback equalizer (DFE) appropriate for channels with long and sparse impulse response (IR) is studied. Such channels are encountered in many high-speed wireless communications applications. It is shown that, in cases of sparse channels, the feedforward and feedback (FB) filters of the DFE have a particular structure, which can be exploited to derive efficient implementations of the DFE, provided that the time delays of the channel IR multipath components are known. This latter task is accomplished by other technique, which estimates the time delays based on the form of the channel input-output cross-correlation sequence in the frequency domain. A distinct feature of the resulting DFE is that the involved FB filter consists of a reduced number of active taps that implies some computational savings than conventional DFE. The resulting DFE also exhibits, improved tracking capabilities and faster convergence as compared with the conventional DFE, that implies a shorter training sequence . Moreover, the new algorithm has a simple form and its steady-state performance is almost identical to that of the conventional DFE.

***Index Terms*— Adaptive equalizers, decision feedback equalizers (DFEs), multipath channels.**

I. INTRODUCTION

In many wireless communication systems the involved multipath channels exhibit a long time dispersion, and delay spreads of up to 40 μs are often encountered. A typical application of this is high definition television (HDTV) signal terrestrial transmission, where the involved channels consist of a few non-negligible echoes, some of which may have quite large time delays with respect to the main signal (see for instance the HDTV test channels reported in several ATSC documents and summarized in [1]). If the information signal is transmitted at high symbol rates through such a dispersive channel, then the introduced intersymbol interference (ISI) has a span of several tens up to hundreds of symbol intervals. This in turn implies that quite long adaptive equalizers are required at the receiver's end in order to reduce effectively the ISI component of the received signal. Note that the situation is even more demanding whenever the channel frequency response exhibits deep nulls.

The adaptive decision feedback equalizer (DFE) has been widely accepted as an effective technique for reducing ISI [2], [3]. Moreover, it has been shown that the DFE structure is particularly suitable for multipath channels, since most part of ISI is due to the long postcursor portion of the impulse response (IR). Recall that an important feature of the DFE is that the postcursor ISI is almost perfectly cancelled by the feedback (FB) filter, provided of course that the previous decisions are correct. Since the postcursor ISI is cancelled by the FB filter, a relatively shorter feedforward (FF) filter is adequate to reduce the remaining ISI. Moreover since noise is involved only in the output of the FF filter, the DFE exhibits less noise enhancement effects as compared with linear equalizers. In high-speed wireless applications, of the type described above, the implementation of a DFE algorithm becomes a difficult task for two main reasons. First, due to the small intersymbol interval, the time available for real-time computations is very limited. Second, due to the long span of the introduced ISI, the DFE must have a large number of taps, which implies heavy computational load per iteration.

During the last decade there have been many efforts in different directions toward developing efficient implementations of the DFE. As such directions, we mention IIR methods, block adaptive implementations, efficient algebraic solutions, modified DFE schemes, etc. [4]–[12]. As mentioned above, in the applications of interest, the involved multipath channel has a discrete sparse form. Efficient DFE schemes which exploit the sparseness of the channel IR have been derived in [13]–[15].

In this paper, a new DFE algorithm, appropriate for sparse multipath channels is studied, proposed in [19]. The algorithm consists of two steps. In the first step, the time delays of the multipath components are estimated in a novel way by properly exploiting the channel IR form [16]. In the second step, the DFE is applied, with the FB filter having a significantly reduced number of taps. These taps are selected so as to act only on time positions associated with the estimated time delays of the involved multipath components. A distinct feature of the novel approach followed in this paper is that the required channel parameters are the locations of the multipath components. This is opposed to most of the existing works [14], [15], in which the whole channel IR has to be initially estimated. Moreover, the relation between the active FB tap positions and the echo time delays is

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determined by investigating the special structure of the FF and FB filters in cases of sparse channels.

The main advantages of this algorithm with respect to the conventional DFE are its lower complexity, faster convergence, and improved tracking capabilities. It is important to note that due to the faster convergence the proposed algorithm requires a shorter training sequence as compared with classical DFE, thus, it offers an additional saving in bandwidth. Note that its overall complexity is of the order of the number of multipath components and hence it is, in practice, several times lower as compared with the conventional DFE.

The paper is outlined as follows. In Section II, the multipath channel is described and the problem is formulated. In Section III, the proposed efficient method for estimating and tracking the time delays is presented. The new DFE algorithm is developed in Section IV and relevant computational issues are discussed. In Section V, the new algorithm is tested and some indicative experimental results are provided. Section VI concludes the work.

II. PROBLEM FORMULATION

In this section, we first formulate the problem of information transmission through a multipath channel, and we recall the conventional and well-studied DFE structure which is our starting point for the derivation of the new equalization technique. The notation used throughout the paper is as follows. $x(n)$ denotes a scalar sample or symbol of sequence $\{x\}$ at time n, $c_i(n)$ is the coefficient of filter c at time n, and $X(k)$ is the k-th frequency bin of the discrete Fourier transform (DFT) of a sequence related to $\{x\}$. Finally, vectors and matrices are denoted as lower case bold roman type and as upper case slanted, respectively.

A. Baseband Multipath Channel

The multipath channel is encountered in almost all wireless communication systems, however, its particular form is highly dependent on the specific system and the application environment (i.e., bit rate, modulation type, carrier frequency, transmitter–receiver separation and the around topography, cell type - if it is for a cellular system - and the receiver's motion within the cell, etc.).

In general, the baseband IR of a multipath channel with discrete components is written as [3]

$$h_c(t,\tau)=\sum_i \alpha_i(t,\tau)e^{j(2\pi f_c\tau_i(t)+\phi_i(t,\tau))}\delta(\tau-\tau_i) \quad (1)$$

where $\alpha_i(t,\tau)$, $\tau_i(t)$ are the real amplitudes and excess delays, respectively, of the multipath component at time t. The phase term $2\pi f_c\tau_i(t)+\phi_i(t,\tau)$ represents the phase shift due to free space propagation of the i-th multipath component, plus any additional phase shifts which are encountered in the channel. If the channel IR is assumed to be invariant within a small-scale time interval, then (1) can be simplified to

$$h_c(\tau)=\sum_i \alpha_i e^{-j\theta_i}\delta(\tau-\tau_i) \quad (2)$$

The overall channel IR, including the combined transmitter and receiver filters' response, say, $p(\tau)$ can be written as

$$h(\tau)=\sum_i \alpha_i e^{-j\theta_i}p(\tau-\tau_i) \quad (3)$$

As mentioned in the introduction, in this paper, we deal with sparse multipath channels having a relatively long IR. Due to the sparseness of the multipath channel IR and the form of the pulse shaping function $p(\tau)$, which decreases rapidly, the overall symbol spaced channel IR remains sparse and can be expressed as

$$h(nT)=\sum_{l=0}^{L} h_{n_l}\delta(nT-n_lT) \quad (4)$$

where $L+1$ is the number of the dominant IR components appearing at the symbol spaced time instants, h_{n_l} is the complex amplitude of the l-th component, and n_lT its respective delay, with T being the symbol period. Delay n_0 corresponds to the main signal3 (n_0=0), while the remaining ones correspond either to causal $(n_0>0)$ or to anticausal $(n_0<0)$ components. The symbol spaced IR spans k_1 precursor and k_2 postcursor symbols, respectively. That is the symbol spaced channel IR can be written in the vector form

$$\mathbf{h}=\left[h_{-k_1} \cdots h_0 \cdots h_{k_2}\right]^T \quad (5)$$

From the total of the (k_1+k_2+1) IR coefficients only $L+1$ are assumed to be non-negligible, located at the n_l positions.

B. Conventional DFE

Taking into account (4), the sampled output of the multipath channel can be written as follows:

$$x(n)=\sum_{l=0}^{L} h_{n_l}d(n-n_l)+w(n) \quad (6)$$

where $\{d\}$ is an independent identically distributed (i.i.d.) symbol sequence with variance σ_d^2 and $\{w\}$ is zero-mean complex white Gaussian noise uncorrelated with the input sequence, with variance σ_w^2. Note that symbol period T has been omitted for reasons of simplicity. Obviously, $\{x\}$ suffers from intersymbol interference due to the presence of undesired multipath components and in most cases equalization is necessary for reliable reception.

As mentioned in the introduction, the DFE structure is particularly suitable for equalizing multipath channels. The LMS-based adaptive DFE is given by the following set of equations:

$$\hat{d}(n)=\sum_{k=-M+1}^{0} c_k(n)x(n-k)+\sum_{k=1}^{N} b_k(n)\tilde{d}(n-k) \tag{7}$$

$$\tilde{d}(n)=f\{\hat{d}(n)\} \tag{8}$$

$$e(n)=\hat{d}(n)-\tilde{d}(n) \tag{9}$$

$$c_k(n+1)=c_k(n)-2\mu^c x^*(n-k)e(n) \quad k=-M+1,\dots,0 \tag{10}$$

$$b_k(n+1)=b_k(n)-2\mu^b\tilde{d}^*(n-k)e(n) \quad k=0,\dots,N \tag{11}$$

where $\{x\}$ and $\{\tilde{u}\}$ denote the equalizer's input and decision sequences, respectively, c_k are the coefficients of the M-length FF filter, and b_k are the coefficients of the N-length FB filter (N is taken at least equal to the channel span [10]). $f\{\cdot\}$ stands for the decision device function, μ^c, μ^b are the step sizes and * denotes complex conjugation. It is assumed that a training sequence of appropriate length is available ensuring convergence of the equalizer. That is the equalizer operates initially in a training mode and then switches to a decision directed mode. In the following sections, first, a frequency domain procedure is proposed for detecting the time delays of the multipath components of the channel IR. Then, a new efficient DFE structure is derived, which takes advantage of the special properties of the multipath channel.

III. ESTIMATION OF THE ECHO DELAYS

A well-established nonparametric procedure for estimating the time delays of the multipath components is based on a proper cross-correlation of the input symbols with the corresponding channel output samples. In a time domain implementation, the estimation of the cross-correlation sequence for N lags requires $O(N)$ operations per sample. It is shown that, an appropriate frequency domain expression of the cross-correlation sequence can be viewed as a sum of complex harmonics, with the unknown time delays interpreted as frequencies. Thus, to estimate the time delays, we suggest an FFT-based scheme of complexity $\log(N)$ per sample. The proposed scheme stems from an appropriate partitioning of both channel input and output sequences and is described below.

Let us first formulate the following 2N-DFT sequences for $k=0,1,\dots,2N-1$

$$D(k)=\sum_{m=p}^{N+p-1} d(n+m)e^{-jmk\frac{2\pi}{2N}} \tag{12}$$

$$X(k)=\sum_{m=0}^{2N-1} x(n+m)e^{-jmk\frac{2\pi}{2N}} \tag{13}$$

where p is assumed to be an overestimated value of the noncausal size of the channel IR (i.e $p>k_1$). The same is presumed for the quantity N-p as far as the size of the causal part of the channel IR is concerned. If these facts hold true, the method which is described below detects the positions of all precursor and postcursor components. Note that $X(k)$ in (13) is based on a 2N-length output sequence, while $D(k)$ in (12) results from an N-length input sequence padded with zeros. As it will become evident from the subsequent derivation, this is done in order for all samples of the cross-correlation sequence to be equally weighted. Indeed, if we consider the expected value of the product of the above sequences, we obtain

$$\mathrm{E}\{X(k)D^*(k)\}=\sum_{i=0}^{2N-1}\sum_{m=p}^{N+p-1}\mathrm{E}\{x(n+i)d^*(n+m)\}e^{-j(i-m)k\frac{2\pi}{2N}} \tag{14}$$

where $\mathrm{E}\{\cdot\}$ denotes the expectation operator. If we now substitute(6) to (14), we get

$$\mathrm{E}\{X(k)D^*(k)\}=\sum_{l=0}^{L}h_{n_l}\cdot\left(\sum_{i=0}^{2N-1}\sum_{m=p}^{N-p-1}\mathrm{E}\{d(n+i-n_l)d^*(n+m)\}e^{-j(i-m)k\frac{\pi}{N}}\right) \tag{15}$$

Since p is larger than the noncausal part of the channel IR, it is easily shown that for every l the indices of d and d^* in (15) are identical for N combinations of m and i (with $m=i-n_l$). Therefore, due to the i.i.d. property of the input sequence (15) is written as

$$\mathrm{E}\{X(k)D^*(k)\}=N\sigma_d^2\sum_{l=0}^{L}h_{n_l}e^{-jn_lk\frac{\pi}{N}} \tag{16}$$

for $k=0,1,\dots,2N-1$. That is, we end up with a sum of complex harmonics at normalized frequencies $\frac{n_l}{2N}$. Applying the 2N - IDFT to the resulting sequence, the locations n_l of the multipath components are determined at the nonnegligible points of the IDFT.

Obviously, in a practical situation, time averaging is used instead of $E\{\cdot\}$ in order to implement (16). In cases where the channel is assumed stationary, the above procedure can be done once during the training phase and then the obtained time delays can be used in the algorithm as described in the next section. Of course, in most situations in practice, the channel exhibits variations and, thus, the required time delays have to be tracked continuously. During tracking, the frequency domain expression of the cross-correlation sequence is formed using the decisions provided by the equalizer (which operates in a decision directed mode).

Exponentially fading memory is imposed on the estimation procedure by including a forgetting factor λ in the frequency domain expression of the cross-correlation sequence as follows:

$$C_{DX}^{(R)} = \sum_{r=0}^{R-1} \lambda^{(R-1-r)N} R_r(k) D_r^*(k) \qquad (17)$$

where $0 \ll \lambda < 1$ and

$$D_r(k) = \sum_{m=p}^{N+p-1} \lambda^{N+p-m-1} d(n+rN+m) e^{-jmk\frac{2\pi}{2N}} \qquad (18)$$

$$X_r(k) = \sum_{i=0}^{2N-1} x(n+rN+i) e^{-jki\frac{2\pi}{2N}}$$

for $k = 0,1,\ldots,2N-1$. Note that if factor λ were included only in (17), then the exponential weighting would be applied on a block-by-block basis, thus affecting the tracking capabilities of the new algorithms. However, additionally including λ in (18) is equivalent to applying an exponential window in the time-domain sample-by-sample computation of the cross-correlation lags. When a new N-length block of input and output samples is available, $C_{DX}^{(R)}(k)$ is updated as

$$C_{DX}^{(R+1)} = \lambda^N C_{DX}^{(R)} + R_R(k) D_R^*(k) \qquad (19)$$
$$k = 0,1,\ldots,2N-1$$

Recall that quantity $C_{DX}^{(R)}(k)$ can be interpreted as a sum of complex harmonics with unknown frequencies and complex amplitudes. Indeed, as can be easily seen by inspecting (16), the frequency bin k corresponds to the sequence index while the time delay n_l corresponds to the unknown frequency.

Determination of Dominant Components: We see from (17) that samples $R{\cdot}N$ of $\{x\}$ and $\{d\}$ are used to compute $C_{DX}^{(R)}(k)$. The L+1 IDFT points of (17) having the highest amplitude are then chosen as the desired locations. The number L of the dominant undesired can be computed by setting a threshold and select the locations of the IDFT points of (17) having amplitudes which exceed this threshold.

IV. THE NEW METHOD

In the proposed algorithm, we focus our attention to the demanding FB part and reduce the computational load by properly selecting $O(L)$ number of taps out of N taps. The main idea behind the derivation of the algorithm is that due to the channel sparseness, the FB filter also possesses a specific sparse form. After exploiting its sparse form, the FB filter is built so as to act only to a restricted set of tap positions. As a result, the algorithm offers significant computational savings while its steady-state error performance is similar to that of the conventional DFE.

A. Derivation of the Algorithm

It is well known [2], [4] that in the minimum mean-squared error (MMSE) DFE, the FF and FB coefficients can be expressed in terms of the channel IR coefficients. Indeed, based on the assumption that previously detected symbols are correct, the minimization of the mean-squared error (MSE) $E\{|e(n)|^2\}$ leads to the following set of equations for the FF filter $\mathbf{c}_M$ and the FB filter $\mathbf{b}_N$

$$\mathbf{c}_M = \left(\mathbf{H}_1 \mathbf{H}_1^H + \frac{\sigma_w^2}{\sigma_d^2} \mathbf{I}_M \right)^{-1} \mathbf{H}_1 \mathbf{e}_{M+k_1} \qquad (20)$$

$$\mathbf{b}_N = \begin{bmatrix} -\mathbf{H}_2^H \mathbf{c}_M \\ \mathbf{0}_{(N-k_2)\times 1} \end{bmatrix} \qquad (21)$$

where $(\cdot)^H$ stands for the conjugate transpose operation, $\mathbf{I}_M$ is the $M \times M$ identity matrix, $\mathbf{e}_{M+k_1} = [0\ \ldots\ 0\ 1]^T$ and the $M \times (k_1 + M)$, $M \times k_2$ matrices are given as shown in (22) and (23),

$$\mathbf{H}_1 \equiv [\mathbf{H}_{11} \mid \mathbf{H}_{12}] = \left[\begin{array}{cccc|cccccc} h_{-k_1} & h_{-k_1+1} & \cdots & h_{-1} & h_0 & h_1 & \cdots & \cdots & h_{M-2} & h_{M-1} \\ 0 & h_{-k_1} & \cdots & h_{-2} & h_{-1} & h_0 & \cdots & \cdots & h_{M-3} & h_{M-2} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & h_{-k_1} & h_{-k_1+1} & \cdots & \cdots & \cdots & \cdots & h_{M-k_1} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 & \cdots & h_{-k_1} & \cdots & h_{-1} & h_0 \end{array}\right] \tag{22}$$

$$\mathbf{H}_2 = \begin{bmatrix} h_M & h_{M+1} & \cdots & h_{k_2} & 0 & \cdots & 0 \\ h_{M-1} & h_M & \cdots & \cdots & \cdots & \cdots & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ h_1 & h_2 & \cdots & \cdots & \cdots & \cdots & h_{k_2} \end{bmatrix} \tag{23}$$

It is shown in [20], that for the special class of channels we consider, and for medium or high SNRs, the solution of (20) can be approximated very closely by the solution of the much more simple set of equations.

$$\mathbf{H}_{12}^H \hat{\mathbf{c}}_M = \mathbf{e}_M \tag{24}$$

where $\mathbf{e}_M = [0 \; \ldots \; 0 \; 1]^T$. Obviously, to solve the above system the condition that the $M \times M$ matrix $\mathbf{H}_{12}^H$ is nonsingular is required.

1) Solution of $\mathbf{H}_{12}^H \hat{\mathbf{c}}_M = \mathbf{e}_M$: From (22), $\mathbf{H}_{12}^H$ can be written as

$$\mathbf{H}_{12}^H = h_0^* \mathbf{I}_M + \mathbf{F} = h_0^* \left(\mathbf{I}_M + h_0^{*-1}\mathbf{F}\right) \tag{25}$$

where F results from $\mathbf{H}_{12}^H$ after removing its main diagonal, i.e.,

$$\mathbf{F} = \begin{bmatrix} 0 & h_{-1}^* & \cdots & h_{-k_1}^* & \cdots & 0 \\ h_1^* & 0 & \cdots & h_{-k_1+1}^* & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & h_{-k_1}^* \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ h_{M-2}^* & h_{M-3}^* & \cdots & \cdots & 0 & h_{-1}^* \\ h_{M-1}^* & h_{M-2}^* & \cdots & \cdots & h_1^* & 0 \end{bmatrix} \tag{26}$$

By assuming that $\|\mathbf{F}\| < |h_0|$, where $\|\cdot\|$ stands for any matrix norm, we can express $\hat{\mathbf{c}}_M$ by means of a Taylor series expansion as follows :

$$\begin{aligned} \hat{\mathbf{c}}_M &= \left(\mathbf{H}_{12}^H\right)^{-1} \mathbf{e}_M \approx \\ &\approx h_0^{*-1}\left(\mathbf{I}_M - h_0^{*-1}\mathbf{F} + h_0^{*-2}\mathbf{F}^2\right)\mathbf{e}_M \end{aligned} \tag{27}$$

where up to second-order terms have been kept in the expansion. Due to the sparseness of the channel IR and the form of matrix F, $\hat{\mathbf{c}}_M$ can be directly expressed in terms of the nonzero coefficients of the channel IR. More specifically, from (27) and the definition of $\mathbf{e}_M$, we easily derive the following results concerning zeroth-, first-, and second-order terms of $\hat{\mathbf{c}}_M$, respectively.

- There exists a zeroth-order contribution, equal to h_0^{*-1}, to the last element of $\hat{\mathbf{c}}_M$.
- For each $n_i < 0$, there is a first-order contribution, equal to $-h_0^{*-2} h_{n_i}^*$, to the $(M + n_i)$th element of $\hat{\mathbf{c}}_M$. This is obvious if we see that $\mathbf{F}\mathbf{e}_M$ is in fact the last column of F.
- For each combination of n_i, n_j with $n_i + n_j < 0$ there is a second-order contribution, equal to $-h_0^{*-3} h_{n_i}^* h_{n_j}^*$, to the $(M + n_i + n_j)$th element of . $\hat{\mathbf{c}}_M$ This is shown by forming the product of F with its last column $\mathbf{F}\mathbf{e}_M$ and taking into account the positions of the nonzero elements of the channel IR.

In conclusion, vector $\hat{\mathbf{c}}_M$ can be expressed up to second-order approximation as follows:

$$\begin{aligned} \hat{\mathbf{c}}_M \approx\; & h_0^{*-1}\mathbf{e}_M^{(M)} - h_0^{*-2} \sum_{n_i<0} h_{n_i}^* \mathbf{e}_M^{(M+n_i)} + \\ & + h_0^{*-3} \sum \sum_{n_i + n_j < 0} h_{n_i}^* h_{n_j}^* \mathbf{e}_M^{(M+n_i-n_j)} \end{aligned} \tag{28}$$

where $\mathbf{e}_M^{(k)} = [0 \cdots 0 \; 1 \; 0 \cdots 0]^T$ of length M has one in its k-th position. Following the analysis of the Appendix, we deduce that the FF filter $\mathbf{c}_M$, computed exactly via (20), can also be expressed by (28) with a very high degree of precision.

2) The FB filter $\mathbf{b}_N$: The form of the FB filter $\mathbf{b}_N$ can be now obtained by combining (21), (23), and (28). More specifically, from (23) and (28) and the result of the Appendix, we get

$$\begin{aligned} \mathbf{H}_2^H \mathbf{c}_M \approx\; & h_0^{*-1}\mathbf{h}_2^{(M)} - h_0^{*-2} \sum_{n_i<0} h_{n_i}^* \mathbf{h}_2^{(M+n_i)} + \\ & + h_0^{*-2} \sum \sum_{n_i + n_j < 0} h_{n_i}^* h_{n_i}^* \mathbf{h}_2^{(M+n_i+n_j)} \end{aligned} \tag{29}$$

where $\mathbf{h}_2^{(k)}$ stands for the k-th column of $\mathbf{H}_2^H$. Since for each l with $n_l > 0$, $h_{n_l}^*$ is the n_l-th element of the last column of $\mathbf{H}_2^H$ and matrix $\mathbf{H}_2^H$ has a Toeplitz form, we deduce from (21) and (29) that the FB filter possesses approximately the following structure.

1) There are first-order ("primary") nonzero taps at the positions n_l where $n_l > 0$ is a position of a causal component in the channel IR.

2) For each "primary" tap at $n_l > 0$, there are second-order nonzero taps at the positions $n_l + n_i > 0$, where $n_i < 0$ are positions of the anticausal components in the channel IR.

3) For each "primary" nonzero tap at $n_l > 0$, there are third-order terms located at $n_l + n_i + n_j > 0$, where n_i, n_j is any combination of component locations with $n_i + n_j < 0$.

Thus, it turns out that the FB filter has a sparse form and, hence, can be restricted to act to the above positions only. In case strong echoes are not present in the channel IR, a second-order approximation of the FB filter [points 1) and 2) above] seems to be sufficient for the proposed algorithm to achieve a performance similar to that of the conventional DFE. However, when there are strong components in the channel IR (especially strong precursor components), a higher number of taps should be considered for the FB filter as dictated by point 3). This results in a slight increase of the computational complexity of the proposed algorithm. In any case, the FB filter comprises a small number of taps and the novel sparse equalizer offers considerable computational savings compared with the conventional DFE.

B. Complexity Issues

The main feature of the algorithm described in Section IV-A is that instead of a long FB filter, it uses a small number of nonzero FB taps. As a result, it is expected that its computational load will be equally reduced compared with the conventional DFE structure. In Table I, the computational complexity (expressed in number of complex multiplications per sample) of the studied algorithm is compared with that of the conventional DFE, under the assumption that N is a power of two. Both cases of a second [SDFE-(2)] and a third [SDFE-(3)] order approximation of the FB filter are considered as analyzed in Section IV-A2. In Table I, L_1, L_2 correspond to the number of detected causal and noncausal multipath components, respectively. S stands for the number of pairs of locations n_i, n_j for which $n_i + n_j < 0$. It can be verified that both variations of the new DFE have significantly lower computational complexity compared with that of the conventional DFE. This is so because the complexity of the conventional algorithm depends linearly on N, while the complexity of the proposed algorithm depends on $\log_2(N)$.

Table I
Comparison in terms of numbers of complex multiplications

Conventional DFE - LMS	$2M + 2N$
SDFE-2	$2M + 3\log_2(N) + 2L_1(L_2 + 1) + 5$
SDFE-3	$2M + 3\log_2(N) + 2L_1(L_2 + S + 1) + 5$

V. SIMULATION RESULTS

The low complexity DFE algorithms have been tested for different sparse channels (including measured microwave channels) and various noise specifications. Their performance has been evaluated for time invariant channels and also for slow time varying channels. Some simulation result are described below.

Fig. 1 shows a typical terrestrial HDTV channel IR. The channel IR is the convolution of test channel D of [17] with a square-root raised cosine filter with 11.5% rolloff. Note the presence of four postcursor components, including a strong far echo, and one precursor component of relatively low magnitude. The input to the channel is a 16-quadrature amplitude modulation (QAM) sequence, while complex white Gaussian noise is added to the channel output, resulting in an SNR of 25 dB.

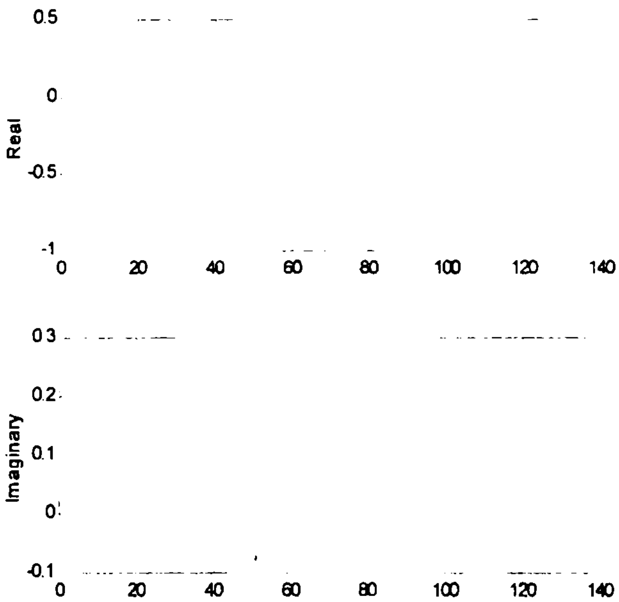

Fig.1. Multipath channel impuls response

In Fig. 2, the performance of low complexity technique (SDFE) is compared with that of the conventional DFE, for channel IR of Fig.1. In the estimation of echo delays algorithm, a threshold of 0,02 has been set for selecting the multipath components with important amplitudes. The number of training symbols is large enough to compare the convergence and steady-state error performance. The red curve correspond to the conventional DFE and the blue and black curves-to the SDFE with second- and third-order approximation of the FF and FB filter, respectively. We see that both SDFEs have greater steady-state performance and faster convergence than conventional DFE with the same length of FF and FB. Also SDFE(3) co..verge to ste..y-st..te f..ster th..n SDFE(2) and depends on the estimation of echo delay algorithm convergence. Figure 3. depictes constellation of received signal (a), and equalized signal employed ....t.... DFE (b), SDFE-2 ( ) .... SDFE-3 ( ) after achieving convergence.

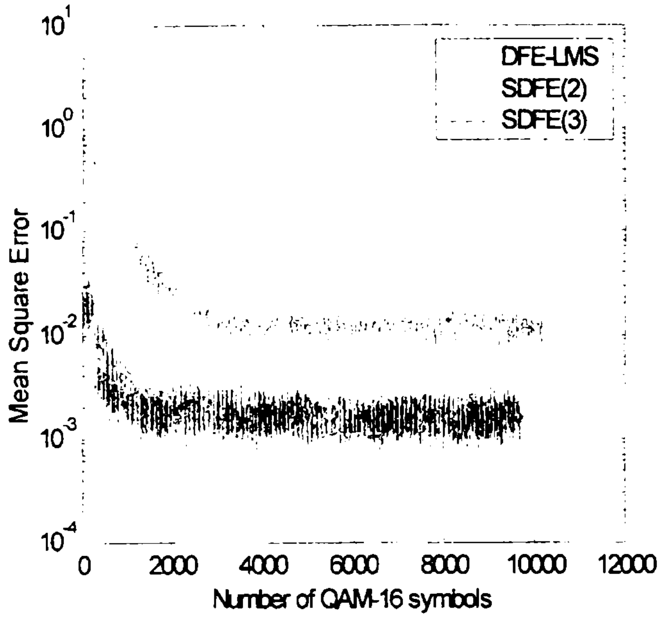

Fig.2. Convergence and steady-state MSE curves of DFE-LMS and SDFEs for the channel IR of Fig.1.

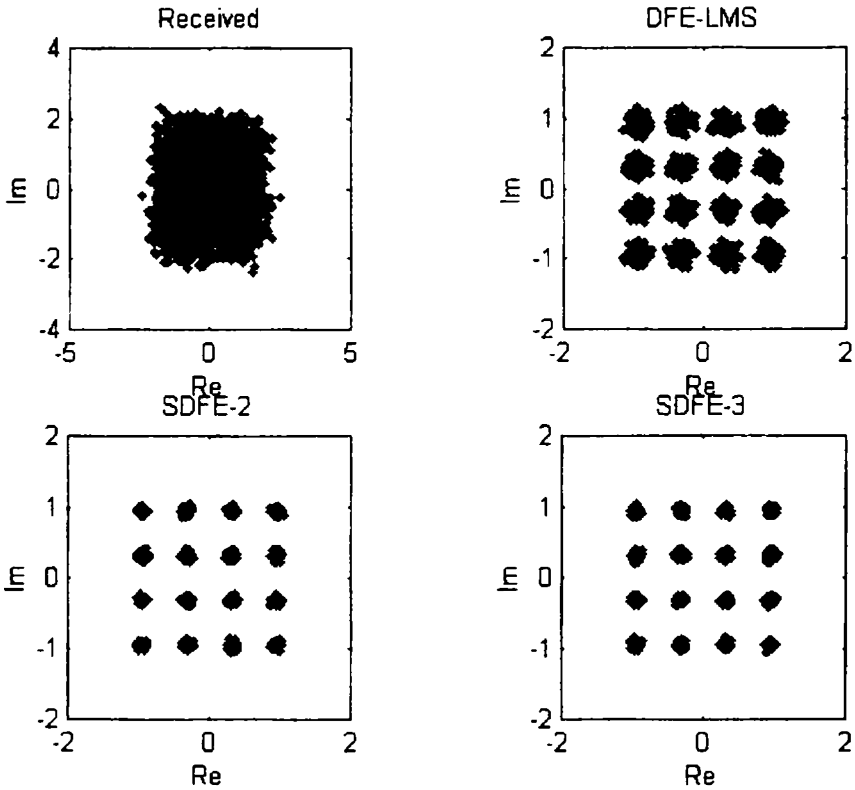

Fig.3. Constelation of received signal (a), equalized signal DFE-LMS (b), SDFE-2 (c), SDFE-3 (d).

In order to investigate the tracking ability of the new algorithm in a time-varying environment, we consider the following scenario. After 7500 iterations, the phases of all postcursor components of the channel of Fig. 1 start continuously rotating, while their amplitudes are kept fixed. The phase rotation step is (0.1/360) rad per iteration. In Fig.4. we see that the new algorithm tracks the change in the environment and as a result the misajustment error is small, lower than in the case of conventional DFE. The convergence and track capability of SDFE depend on the convergence of estimation of echo delay algorithm.

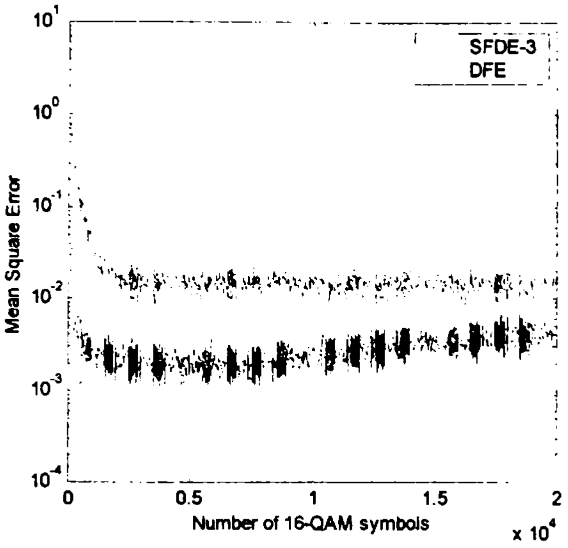

Fig.4. MSE curve under slow time-varying CIR

VI. CONCLUSION

In this paper, the sparse form encountered in many high-speed wireless communications channels is properly exploited in two ways. First, the time delays of the dominant multipath components are efficiently estimated and continuously updated using a frequency domain approach. Second, an approximate form of the FB filter of the MMSE-DFE is derived. Based on the above, a new efficient DFE algorithm is proposed, whose FB filter comprises a reduced number of active taps. Depending on the multipath channel conditions, there may exist a tradeoff between the number of FB taps used and the performance of the proposed algorithm. In general, the new algorithm offers considerable computational savings, faster convergence, and acceptable tracking capabilities while exhibiting almost identical steady-state performance in most practical cases, as compared with the conventional DFE. The features of the new algorithm have been confirmed through extensive simulation tests.

REFERENCES

[1] M. Ghosh, "Blind decision feedback equalization for terrestrial television receivers," *Proc. IEEE*, vol. 86, pp. 2070–2081, Oct. 1998.

[2] J. G. Proakis, *Digital Communications*, 3rd ed. New York: McGraw-Hill, 1995.

[3] T. S. Rappaport, *Wireless Communications: Principles and Practice*.Englewood Cliffs, NJ: Prentice-Hall, 1996.

[4] J. Salz, "Optimum mean-square decision feedback equalization," *BellSyst. Tech. J.*, vol. 52, no. 8, pp. 1341–1373, Oct. 1973.

[5] G. Young, "Reduced complexity decision feedback equalization for digital subscriber loops," *IEEE J. Select. Areas Commun.*, vol. 9, pp.810–816, Aug. 1991.

[6] P. Crespo and M. Honig, "Pole-zero decision feedback equalization with a rapidly converging adaptive IIR algorithm," *IEEE J. Select.Areas Commun.*, vol. 9, pp. 817–829, Aug. 1991.

[7] N. Al-Dhahir and J. M. Cioffi, "Stable pole-zero modeling of long FIRfilters with application to the MMSE-DFE," *IEEE Trans. Commun.*, vol.45, pp. 508–513, May 1997

[8] K. Berberidis, T. A. Rontogiannis, and S. Theodoridis, "Efficient block implementation of the DFE," *IEEE Signal Processing Lett.*, vol. 5, pp.129–131, June 1998.

[9] K. Berberidis and P. Karaivazoglou, "A block adaptive DFE in the frequency domain based on tentative decisions," in *Proc. EUSIPCO*,Rodos, Sept. 1998.

[10] N. Al-Dhahir and J. M. Cioffi, "Fast computation of channel-estimate based equalizers in packet data transmission," *IEEE Trans. Signal Processing*,vol. 43, pp. 2462–2473, Nov. 1995.

[11] S. Ariyavisitakul and L. J. Greenstein, "Reduced-complexity equalization techniques for broadband wireless channels," *IEEE J. Select. AreasCommun.*, vol. 15, pp. 5–15, Jan. 1997.

[12] R. Gupta, Kiran, and E. Lee, "Computationally efficient version of the decision feedback equalizer," in *Proc. IEEE ICASSP*, Seattle, 1999, pp.1257–1260.

[13] S. Ariyavisitakul, N. R. Sollenberger, and L. J. Greenstein, "Tap-selectable decision feedback equalization," *IEEE Trans. Commun.*, vol.45, pp. 1498–1500, Dec. 1997.

[14] I. J. Fevrier, S. B. Gelfand, and M. P. Fitz, "Reduced complexity decision feedback equalization for multipath channels with large delay spreads,"*IEEE Trans. Commun.*, vol. 47, pp. 927–937, 1999.

[15] S. F. Cotter and B. D. Rao, "Matching pursuit based decision-feedback equalizers," in *Proc. IEEE ICASSP*, Istanbul, Turkey, June 2000.

[16] K. Berberidis and A. A. Rontogiannis, "Efficient decision feedback equalizer for sparse multipath channels," in *Proc. IEEE ICASSP*, Istanbul, Turkey, June 2000.

[17] I. Kang, M. P. Fitz, and S. B. Gelfand, "Blind estimation of multipath channels: A modal analysis approach," *IEEE Trans. Commun.*, vol. 47, pp. 1140–1150, Aug. 1999

[18] A. A. Rontogiannis, A. Marava, K. Berberidis, and J. Palicot, "Semiblind estimation of multipath channel parameters via a separable least squares approach," in *Proc. 14th Digital Signal Processing Conf.*, Santorini, July 2002, pp. 49–52.

[19] A. A. Rontogiannis and K. Berberidis, "Efficient decision feedback equalizer for sparse wireless channels," *IEEE Trnas. Wireless Commun.* vol. 2, May 2003.

[20] M. H. Hayes, *Statistical Digital Signal Processing and Modeling*. New York: Wiley, 1996.