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A discrete model for reference source noise in indirect frequency synthesis

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Abstract-Indirect frequency synthesis preserves its actuality and is still offering good possibilities to achieve performing systems. Practical problems refer to obtaining competitive acquisition times, under an intense traffic and noisy environment. The paper is focusing on significant aspects such as transfer functions, signal/noise ratio. This paper presents a brief description of the theoretical principle of these systems, especially those concerning aspects of the noise content in output signal due to reference source. A Matlab support for solutions of the applications and simulation is also used.

Keywords: transfer function, reference source noise.

I. FREQUENCY CHARACTERISTIC OF THE REFERENCE

If we are to analyse the behaviour of an numerical frequency synthesizer when the reference oscillator or voltage controlled oscillator signals are frequency or phase modulated by an useful or noise signal, we must determine frequency and phase characteristics versus the useful or noising signal.

Consider an indirect frequency synthesizer like in fig.1, with frequency variable reference phase $\Delta\theta_i(s)$.

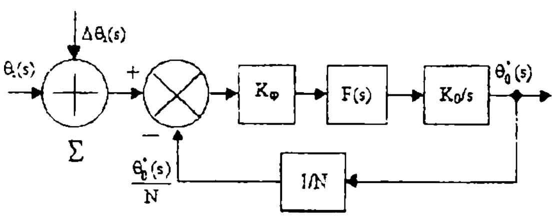

Fig 1 Basic structure of an indirect frequency synthesizer

The transfer function of the reference oscillator noise is:

$$\frac{\Delta\theta_0(s)}{\Delta\theta_i(s)}=\frac{K_0K_\varphi F(s)}{s+\frac{K_0K_\varphi F(s)}{N}}=S_\theta(s) \qquad (1)$$

where:

$\Delta\theta_i(s)$ – Laplace image of input phase variation due to the perturbation;

$\Delta\theta_0(s)$ – Laplace image of the output phase variation due to $\Delta\theta_i(s)$;

$\theta^*_0(s)$ – output signal with perturbations;

K_0[rad/s/V]- the VCO gain;

K_φ[V/rad] – the gain of the phase detector in phase regime ;

F(s)- Laplace transform of the loop filter transfer function;

N – the integer divider ratio of the programmable divider of the loop.

For a type 2 second order loop the function (1) becomes:

$$\frac{\Delta\theta_{0i}(s)}{\Delta\theta_0(s)}=N\cdot\frac{2\xi\omega_n s+\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2} \qquad (2)$$

where:

ξ - damping factor of the loop;

ω_n – natural pulsation of the undamped system.

With the substitution $s = j\omega_m$ in (2) we have the response of the reference oscillator at a noise component of frequency ω_m:

$$\frac{\Delta\theta_{0i}(j\omega_m)}{\Delta\theta_0(j\omega_m)}=N\cdot\frac{2\xi j\omega_n\omega_m+\omega_n^2}{-\omega_m^2+2\xi\omega_n\omega_m+\omega_n^2} \qquad (3)$$

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II. EQUATIONS

If some elements (phase detector, frequency dividers) of the system are digitally ones, a digital approach is possible, so that the noise transfer function is more useful with z transform:

$$\frac{\Delta\theta_{0i}(z)}{\Delta\theta_i(z)} = \frac{n_2 z^2 + n_1 z + n_0}{m_2 z^2 + m_1 z + m_0} \quad (4)$$

The equation with finite differences for the input $\Delta\theta_i(n)$ and output $\Delta\theta_{0i}(n)$ samples is:

$$\begin{aligned}\Delta\theta_{0i}(n) &= \frac{1}{m_2}[n_2 \cdot \Delta\theta_i(n) + n_1 \cdot \Delta\theta_i(n-1) + \\ &+ n_0 \cdot \Delta\theta_i(n-2)] - \frac{1}{m_2}[m_1 \cdot \Delta\theta_{0i}(n-1) + \\ &+ m_0 \cdot \Delta\theta_{0i}(n-2)\end{aligned} \quad (5)$$

With bilinear transform method the transfer function in z is:

$$\frac{\Delta\theta_{0i}(z)}{\Delta\theta_i(z)} = \frac{\left(a_2 + a_1\frac{T}{2} + a_0\frac{T^2}{4}\right)z^2 + \left(-2a_2 + a_0\frac{T^2}{2}\right)z + \left(a_2 - a_1\frac{T}{2} + a_0\frac{T^2}{4}\right)}{\left(b_2 + b_1\frac{T}{2} + b_0\frac{T^2}{4}\right)z^2 + \left(-2b_2 + b_0\frac{T^2}{2}\right)z + \left(b_2 - b_1\frac{T}{2} + b_0\frac{T^2}{4}\right)} \quad (6)$$

where:

$$\begin{cases} a_2 = 0 \\ a_1 = 2\xi\omega_n N \\ a_0 = \omega_n^2 \end{cases} \quad (7)$$

and:

$$\begin{cases} b_2 = 1 \\ b_1 = 2\xi\omega_n \\ b_0 = \omega_n^2 \end{cases} \quad (8)$$

T is the period of the reference signal.
Making the identification between the relations (6),(7) and (8), the coefficients *m* and *n* are:

$$\begin{cases} m_2 = \left(\omega_n \frac{T}{2}\right)^2 + 2\xi\omega_n \frac{T}{2} + 1 \\ m_1 = -2 + \left(\omega_n \frac{T}{2}\right)^2 \\ m_0 = \left(\omega_n \frac{T}{2}\right)^2 - 2\xi\omega_n \frac{T}{2} + 1 \end{cases} \quad (9)$$

respectively:

$$\begin{cases} n_2 = \left[\xi\omega_n T + \left(\omega_n \frac{T}{2}\right)^2\right] \cdot N \\ n_1 = \frac{(\omega_n T)^2}{2} \cdot N \\ n_0 = \left[-\xi\omega_n + \left(\omega_n \frac{T}{2}\right)^2\right] \cdot N \end{cases} \quad (10)$$

The relations (9) and (10) allow the obtaining of the noise levels of the output versus the reference noise, using Simulink models.

III. DIAGRAMS

We realize two situations:
a) $\xi = 0{,}707;\quad N = 1000;\quad \omega_n T = 2\pi \cdot 0{,}1$ (11)
generating the following sets of coefficients *m* and *n*:

$$\begin{cases} m_2 = 1{,}045 \\ m_1 = -1{,}998 \\ m_0 = 0{,}957 \end{cases} \quad (12)$$

$$\begin{cases} n_2 = 45{,}409 \\ n_1 = 1{,}974 \\ n_0 = -43{,}445 \end{cases} \quad (13)$$

The Simulink model is given in fig. 2, and the diagrams of the input/output noise in fig. 3,4, in time and frequency domains, respectively.
The noise is simulated with a gaussian noise generator and we search to observe the response of the system regarding this signal. So we use both a spectral analyser and a oscilloscope to the output.

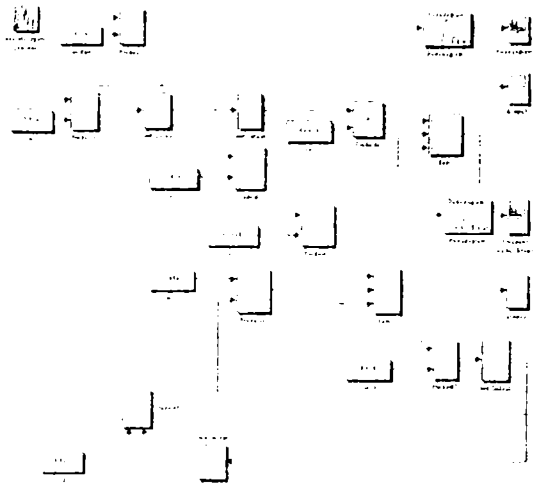

Fig. 2 The Simulink model for $\xi = 0,707$, $N = 1000$, $\omega_n T = 2\pi \cdot 0,01$

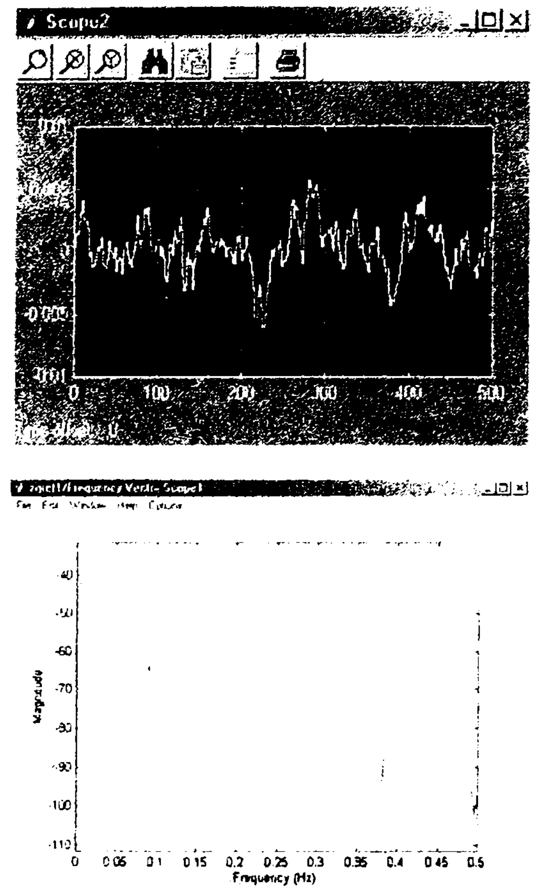

Fig 4 Noise diagrams of the system output in time and frequency domains

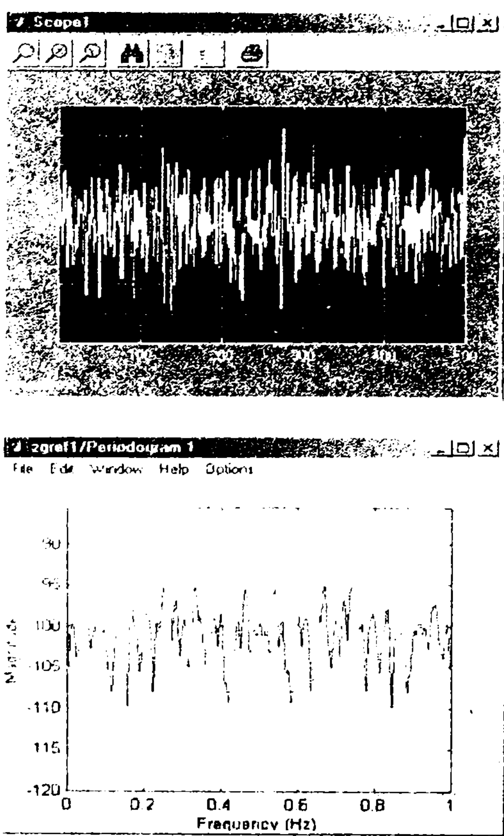

Fig 3 Noise diagrams of the system input in time and frequency domain

b) $\xi = 0,707$, $N = 1000$, $\omega_n T = 2\pi \cdot 0,02$ (14)

The sets for m and n:

$$\begin{cases} m_2 = 1,543 \\ m_1 = -1,803 \\ m_0 = 0,654 \end{cases} \quad (15)$$

$$\begin{cases} n_2 = 542,917 \\ n_1 = 197,392 \\ n_0 = -345,525 \end{cases} \quad (16)$$

The Simulink model and time and frequency diagrams are in fig. 5, 6, and 7, respectively.

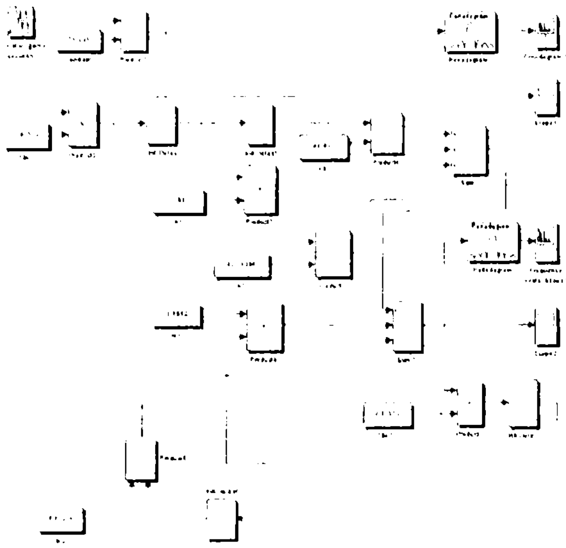

Fig 5 The Simulink model for $\xi = 0{,}707$, N = 1000, $\omega_n T = 2\pi\, 0{,}02$

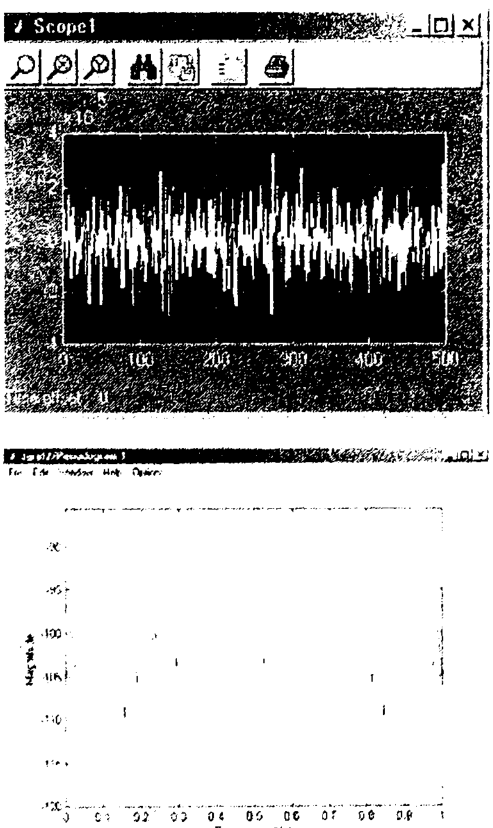

Fig 6 Noise diagrams of the system input in time and frequency domains

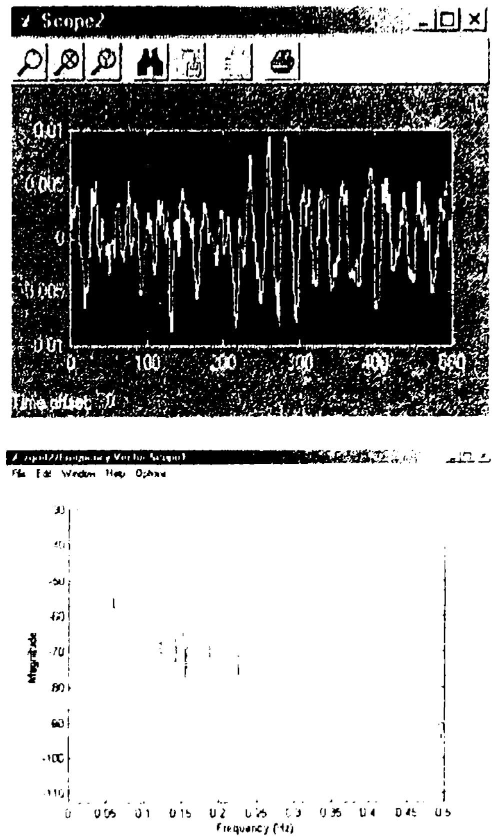

Fig 7 Noise diagram of the system output in frequency domain

IV. CONCLUSIONS

Based on the relations (9) and (10) we calculated the coefficients m_k, n_k ($k = 0,1,2$) of the noise transfer function. These values were introduced in the finite differences equation (5) and this equation was then represented in a Simulink model. We use the value 0,707 for the damping factor ξ (allowing an optimal dinamic response), a value of 1000 for the dividing ratio N of the programmable divider of the loop and the normate frequency $\omega_n T = 2\pi \cdot 0{,}1$ in case a) and $2\pi \cdot 0{,}02$ in case b). The system was tested with Gaussian distributed random input signal.

Analysing the diagrams we can see that the synthesizer has the behaviour of a low-pass filter in the loop in regard to the input noise. The characteristic depends on damping factor ξ of the system and normate frequency $\omega_n T$. The slow phase variations are transmitted to the output.

The synthesizer works like a phase tracking system with gain of N. Because the reference source noise has a contribution in the spectral density of the output noise in [6]:

$$S_{\theta i}(\omega) \cdot N^2 \cdot \left| \frac{2\xi\omega_n s + \omega_n^2}{s^2 + 2\xi\omega_n s + s^2} \right| \qquad (17)$$

where $S_{\theta i}(\omega)$ is the reference source noise spectral density, the conclusion is that despite of the small value, this noise appears multiplied by N^2. If the dividing ratio N is variable (to obtain all the frequencies in the work range) then the lowest frequencies will produce the lowest reference noise effect.

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