

Cartesian coordinate optical filter analyses

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Abstract—Our goal in this paper is to develop a study about how Cartesian coordinate optical filter acts in frequency domain. We start from the amplitude transfer function which is the optical transfer function of filter in this case of illumination. We define different type of filter LPF, HPF and BPF in Cartesian coordinate 2D and 3D then we pass an image through this filter and we see the filter effect on final image function of filter dimension which we used. For this we use an algorithm in three steps: we compute Fourier transform of an image, then we multiply this with filter transfer function then we make inverse Fourier transform on result.

INTRODUCTION

To develop our study we shall recall what is an optical filter?

An optical filter is a 4f telecentric system; this is an aligned system consisting of an input image, first lens, Fourier plane, second lens and output image like in the Fig. 1

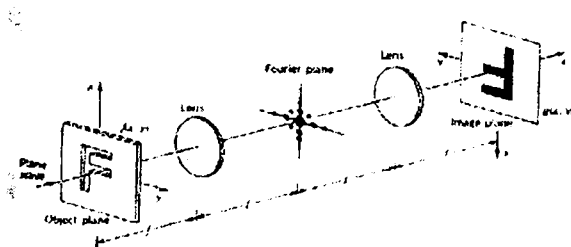


Fig.1 4f system

The aim of lens in such a system is as follows: lens have property of making Fourier transform in optics. If in the Fourier plane we put an aperture (let say for the beginning an arbitrary aperture) then we will have an optical filter like in the Fig. 3 where we can see the difference between filtered image and original image:

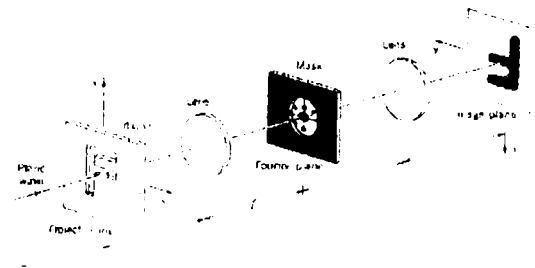


Fig. 2 optical filter

RESULTS OF SIMULATION

In the case of coherent illumination which we use in this paper the amplitude transfer function (see bibl.9) is the filter transfer function. By coherent illumination here we understand parallel light (Fraunhofer diffraction).

Frequency domain method

Let $g(x,y)$ be an image formed by the convolution of an image $f(x,y)$ and a linear, position invariant operator $h(x,y)$ so we have $g(x,y)=h(x,y)*f(x,y)$; then in the frequency domain we will have: $G(u,v)=H(u,v)F(u,v)$;

$H(u,v)$ is optical transfer function and G, H, F are Fourier transforms of g, h, f .

Our goal after we compute $F(u,v)$ is to select $H(u,v)$ so that the desired image $g(x,y)=F^{-1}[H(u,v)F(u,v)]$ to have some new feature of $f(x,y)$.

So in Cartesian coordinate we will define LPF, HPF and BPF.

LPF Low pass filter

A low pass filter has a transfer function like this:

$$H(u,v)=\begin{cases} 1 & \text{if } D(u,v) \leq D_0 \\ 0 & \text{if } D(u,v) > D_0 \end{cases}$$

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D_0 is a specific nonnegative quantity and $D(u,v)$ is

the distance from $H(u,v)$ to the origin of frequency plane. The point of transition between 1 to 0 is called the cutoff frequency; here cutoff frequency is D_0 .

We have the LPF 2D representation in Fig. 3 and 3D representation in Fig. 4



Fig. 3 filter transfer function 2D

LPF have the next characteristics: high frequency reduction but they make blur, noise reduction because noise is installed at high frequency.

Next we will study how LPF acts for different cutoff frequency for an image with 256x256 dimensions we will use the next cutoff dimension: 128x128, 80x80, and 40x40. The result is illustrated Fig. 5

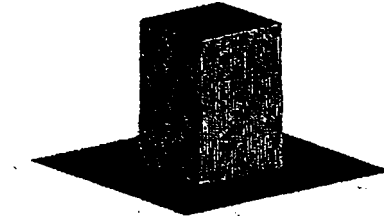


Fig. 4 filter transfer function 3D

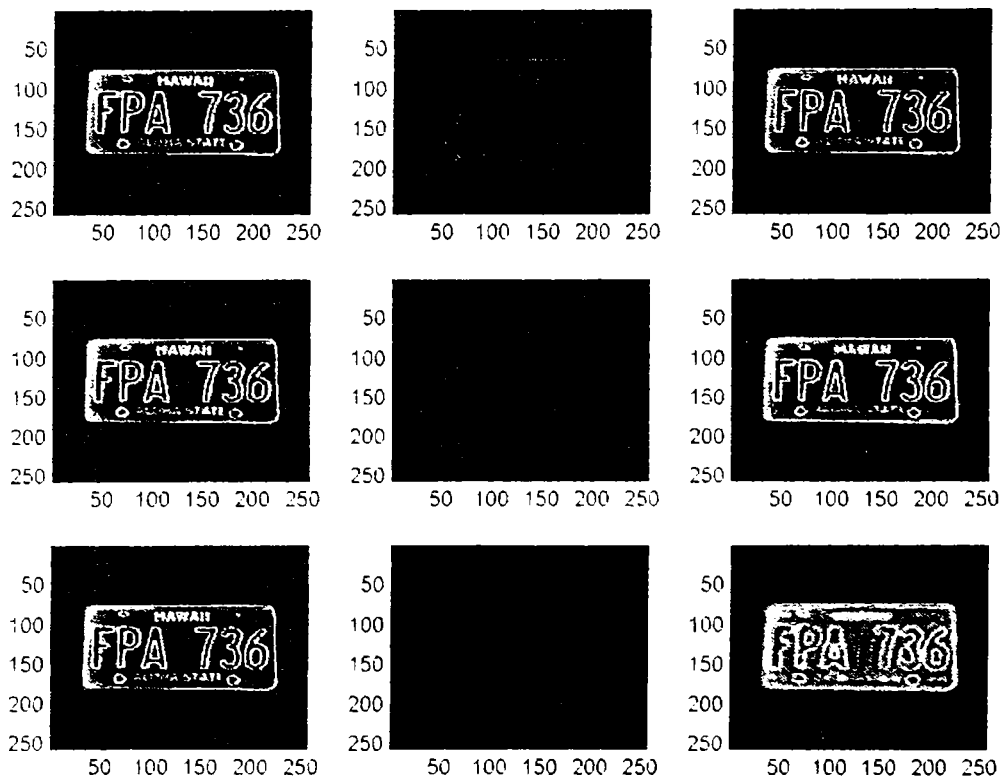


Fig. 5 The graphics present the same image filtered with filters having different cutoff; as the cutoff frequency decrease we will have more blur on the image and oscillation. Oscillations around image are caused by transitory regime passing from 1 to 0.

HPF high pass filter

A HPF has inverse properties like LPF, and is defined like HPF- 1-LPF which we can write like this:

$$H(u,v) = \begin{cases} 0 & \text{if } D(u,v) \leq D_0 \\ 1 & \text{if } D(u,v) > D_0 \end{cases}$$

D_0 is cutoff frequency



Fig. 6 filter transfer function 2D

representation in Fig. 7

HPF has the next characteristics block low pass frequency and let to pass high frequency. As effect it enhances the noise, reduces basic characteristics of an image and is used for edge detections. Next we will study how HPF acts for different cutoff frequency for an image with 256x256 dimensions we will use the next cutoff dimension: 128x128, 80x80, and 40x40. The result is illustrated in Fig. 8.

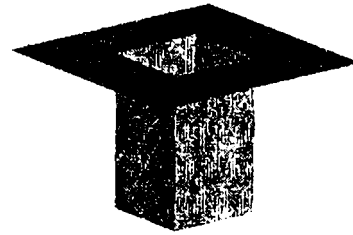


Fig.7 filter transfer function 3D

with the 2D representation in Fig. 6 and 3D

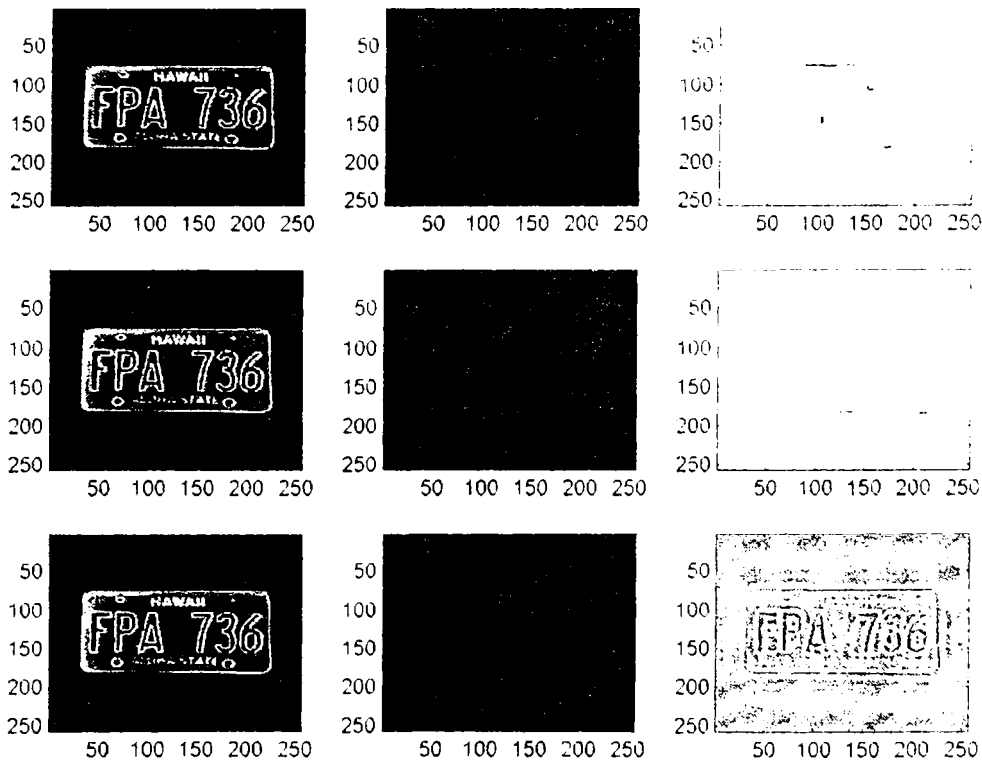


Fig. 8 the graphics present the same image filtered with filters having different cutoff, as the cutoff frequency increase we will have edge detection more pronounced on the image and oscillation. Oscillations around image are caused by transitory regime passing from 1 to 0

BPF high pass filter

A BPF is defined like difference between two LPF

with different cutoff frequency $BPF=H1(u,v)-H2(u,v)$

$$H(u,v) = \begin{cases} 1 & \text{if } D(u,v) \leq D_{01} \\ 0 & \text{if } D(u,v) > D_{01} \end{cases}$$

$$H(u,v) = \begin{cases} 1 & \text{if } D(u,v) \leq D_{02} \\ 0 & \text{if } D(u,v) > D_{02} \end{cases}$$

with the 2D representation in Fig 9 and 3D representation in Fig 10



Fig 9 filter transfer function 2D

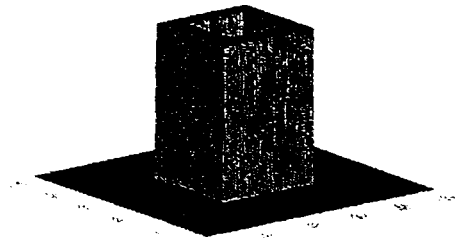


Fig 10 filter transfer function 3D

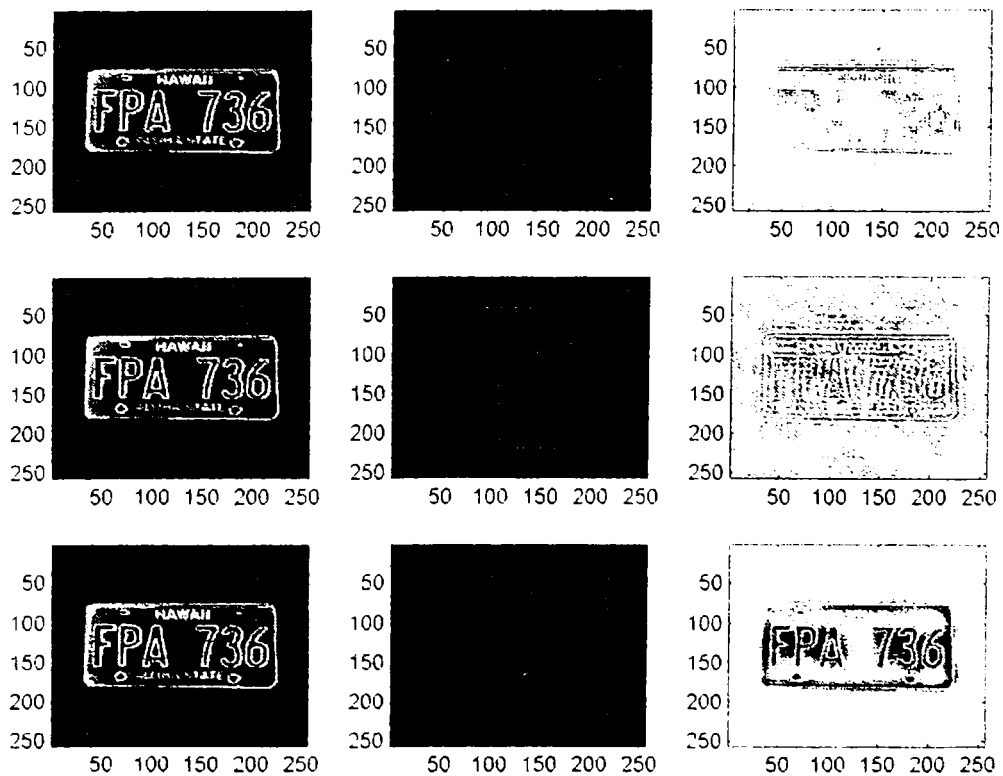


Fig. 11 the graphics present the same image filtered with filters having different dimensions. It keeps specific characteristics of L.PF and HPF and is a mediator filter between this two filter

CONCLUSION

This paper tries to distinguish with concrete example basic characteristics of LPF, HPF and BPF in Cartesian coordinate. These filters are ideals because are defined like 1 and 0 but they play an important role because they describe how an image is filtered in low, high and medium spatial frequency domain. Because ideal character of this filter we observe oscillation around image, oscillation specific transitory regime. There are other optical filters like Gaussian, Hamming, and Butterworth which have a better transitory regime. Optical filter in Cartesian coordinate are very important in Optical Fourier signal and image processing.

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