

## Optimal Algorithms for Scheduling Uplink Users Transmissions in CDMA Systems

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**Abstract:** Early work on capacity analysis of CDMA systems promise a large increase in cellular capacity when compared to other multiple-access techniques. The uplink capacity is shown to be limited by the outage probability. In this work, we consider the problem of optimal scheduling of uplink user transmissions in a single CDMA cell. We assume that the system operates in a TD/CDMA manner, with time-slotted scheduling of transmissions, assisted by periodic feedback of channel and/or congestion information through control channels. The major observation arising from our analysis is that it is advantageous on the uplink to schedule "strong" users one-at-a-time, and "weak" users in larger groups. This contrasts with the downlink where one-at-a-time transmission for all users has shown to be the preferred mode in much previous work.

**Key words:** Scheduling, Uplink, Reverse Link, CDMA, SIR, "strong" users, "weak" users.

### 1. INTRODUCTION

Code division multiple access (CDMA) has received a great deal of attention as a multiple-access method for future mobile networks. Its main advantages are higher radio capacity and capability of flexible data transmission. CDMA was a key technology in the proposals submitted to the International Telecommunications Union (ITU) on Universal Mobile Telecommunications System/International Mobile Telecommunications in the year 2000 (UMTS/IMT-2000). The Association for Radio Industry and Business (ARIB) in Japan became the driving force behind a third-generation radio-transmission technology known as wide-band CDMA (W-CDMA). The European Telecommunications Standards Institute (ETSI) and ARIB have managed to merge their technical proposal into one harmonized WCDMA standard air interface. In the United States, The Telecommunications Industry Association (TIA) has proposed two air-interface standards. One of them is a CDMA-based

interface, referred to as cdma2000, which maintains backward compatibility with existing IS-95 networks. South Korean Telecommunications Technology Association (TTA) supports two air-interface proposals, one similar to WCDMA and the other to cdma2000.

Data scheduling in wireless networks is a widely studied topic due to the impending explosion of high speed wireless data services in third generation (3G) systems. For natural reasons associated with the expected traffic characteristics, most of the previous research has focused on the forward-link/downlink, i.e. base to mobile communication. The traffic is expected to be dominated by web browsing and file downloads. As a result, current wireless data systems employ highly asymmetric link designs (e.g. HDR) with skinny uplinks and fat downlink pipes. However, it has also often been pointed out that there could be a proportional increase in reverse-link/uplink traffic in the form of acknowledgements, feedback etc. along with the growth of other services like ftp, image/data uploads etc. which require high data rates on the uplink. These considerations have resulted in some research on the subject of uplink scheduling, although small compared to the literature on downlink scheduling.

The optimal schedules, as well as our analyzed approximations, seem to possess an intuitively appealing property that is consistent with traditional observations about wireless transmission: *it is advantageous for users with weak channels to transmit simultaneously, and for users with strong channels to transmit one-at-a-time*. Hereafter, we refer to users with low received power at the base even when transmitting at peak transmit power as "weak" users, and the strongly received users at the base as "strong" users. The intuition behind this is that the added interference at the base from simultaneous transmissions of weakly received users to each other

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is small compared to the extraneous interference, and thereby does not affect the user SIRs and data rates significantly. On the contrary, for users received strongly at the base, the penalty in terms of SIR and data rate with simultaneous transmission can be quite significant. The rest of the presentation begins with the problem formulation, followed by a discussion of solution methods, then by approximate scheduling algorithms derived from the optimal solution and related bench-marks, and finally simulation results.

## II. UPLINK SCHEDULING

Consider the uplink of a single CDMA cell serving  $N$  users. Let  $P_i$  be the instantaneous received power and  $g_i$  the SIR of the  $i$ -th user. For simplicity, we express  $P_i$  in units of the total interference  $I+s$ , where  $s$  is thermal noise and  $I$  is the total instantaneous interference from other sources, such as other cells in the network. In order to meet the SIR requirement of all users, we must then have for each  $i$

$$\frac{P_i}{\sum_{j \in \{1, \dots, N\}, j \neq i} P_j} + 1 \geq \gamma_i \quad (1)$$

The feasible SIR vectors  $g$  specified by (1) above has been derived in many previous papers, and we recall it below to point out the specific aspects utilized later in our scheduling algorithm. Given the peak received power of the  $i$ -th user  $P_i$ , computed using the path gain  $G_i$  and peak transmit power  $P_i^t$  as  $P_i = P_i^t G_i$ , we may change variables to  $q_i = P_i^t / P_i^t$  to rewrite (1) as

$$\frac{\theta_i \bar{P}_i}{\sum_{j \in \{1, \dots, N\}, j \neq i} \theta_j \bar{P}_j + 1} \geq \gamma_i \quad (2)$$

A given SIR vector  $g_i$  is feasible if (2) can be satisfied with equality with  $0 \leq q_i \leq 1$  for all  $i$ . We hence examine the solution to the set of linear equations.

$$\frac{\theta_i \bar{P}_i}{\gamma_i} = \sum_{j \in \{1, \dots, N\}, j \neq i} \theta_j \bar{P}_j + 1 \quad (3)$$

which can be further rewritten as

$$\theta_i \bar{P}_i \left(1 + \frac{1}{\gamma_i}\right) = \sum_{j \in \{1, \dots, N\}} \theta_j \bar{P}_j + 1 \quad (4)$$

It can be seen by inspection that the solution is of the form  $q_i P_i (1 + 1/\gamma_i) = C$  where  $C$  is a global parameter. The value of  $C$  can be obtained by substituting the postulated solution in (4) to obtain

$$C = C \sum_j \frac{\gamma_j}{\gamma_{j+1}} + 1 \text{ which gives the final solution}$$

$$\theta_i = \frac{\gamma_i}{\bar{P}_i (1 + \gamma_i) \left[1 - \sum_{j \in \{1, \dots, N\}} \frac{\gamma_j}{1 + \gamma_j}\right]} \quad (5)$$

Defining  $a_i = g_i / (1 + g_i)$ , we see that

$$\theta_i = \frac{\alpha_i / \bar{P}_i}{1 - \sum_j \alpha_j} \quad (6)$$

Clearly,  $0 \leq a_i \leq 1$ . Since we require  $0 \leq q_i \leq 1$ , equations (6) result in the following feasibility conditions to meet the required SIRs.

$$\sum_j \alpha_j + \frac{\alpha_i}{P_i} \leq 1, \forall i \quad (7)$$

Note the simple linear form of the feasible SIRs (7) in terms of the  $a_i$ , about which we make the following observations:

- When there is no power limitation, i.e.  $P_i$  are arbitrarily large for each  $i$ , equations (7) collapse into the single condition  $\sum_j \alpha_j \leq 1$ , which is the

simple, single-cell version of the well-known stability condition for uplink power control. In this case, as we later show, the feasible SIR region has a fully concave boundary which is dominated by its convex hull composed by time-sharing single user transmissions.

- When the power limitations are severe, i.e.  $P_i$  are small for each  $i$ , the constraints (7) approach independent box constraints  $0 \leq a_i \leq P_i$  for each  $i$ , and simultaneous transmission is favored.

- For intermediate cases, where some  $P_i$  are large and others small, the optimal scheduling strategy involves time-sharing over different subsets of simultaneously transmitting users. An interesting observation regarding the optimal strategy, that we prove below, is that all transmitting users would always transmit at full power, i.e. at  $P_i$ .

In the subsequent section, we analyze the last item above, which is the most likely practical scenario, to determine the optimal transmitting sets.

A version of the algorithm that guarantees *queue stability*, i.e. boundedness of queue lengths when feasible, is specified as the rate choice that satisfies

$$R^* = \text{argmax } QR \quad (8)$$

where,  $R, Q$  are rate and queue vectors of the user set respectively, and is the *rate region*, or the set of feasible rate vectors. In general,  $Q$  may be replaced by other choices of *weights* to satisfy other QoS criteria such as delay violation probability, packet loss probability etc. Minimum instantaneous rate guarantees may be satisfied by restricting the rate region appropriately.

Thus, the general optimal scheduling problem can be solved if one has a technique to solve for  $R^* = \text{argmax } wR$  for arbitrary given weights.

To formulate (8) for uplink CDMA scheduling, we require a relationship between rate and SIR as  $R=f(g)$  for each user. We assume this relationship to be *concave* in the argument  $g$ , as is the case for the Shannon formula for the AWGN Gaussian channel where  $R=\text{blog}(1+g)$ . Since (7) are linear in  $a=g/(1+g)$ , it is more convenient to consider the  $R,a$  relationship  $R=g(a)$  which is now *convex* for the Shannon formula as  $g(a)=\text{blog}([1/1-a])$ , (8) then becomes the following optimization problem:

$$\max \sum_{i=1}^N w_i g_i(\alpha_i) \quad (9)$$

subject to

$$\sum_{j=1}^N \alpha_j + \frac{\alpha_i}{P_i} \leq 1, \alpha_i \geq 0, \forall i \quad (10)$$

Typically  $g_i(\cdot)=g(\cdot)$  are identical functions, as is the case for the Shannon formula, but our results remain unaffected even if they were all different, as long as they stay *convex*.

We now present the solution to (9), (10), which determines the subset  $S, S'$  for each scheduling

interval. Defining  $\Lambda = \sum_{j=1}^N \alpha_j$ , we rewrite (9), (10) as

$$\max \sum_{i=1}^N w_i g_i(\alpha_i)$$

subject to

$$\sum_{j=1}^N \alpha_j = \Lambda \quad 0 \leq \alpha_i \leq \bar{P}_i(1-\Lambda), \forall i \quad (11)$$

A simple algorithm can be defined as:

1) Compute, and sort in increasing order, the  $L_i$ , satisfying

$$v_i = v_j \text{ for some } i, j \in \{1 \dots N\}, L_j \in (0, 1)$$

Denote this list as  $\{L_k: m \in \{1, \dots, M\}\} = \{0, L_1, \dots, 1\}$

2) For each interval  $[L_m, L_{m+1}]$ , determine the user ordering in some interior point, say the midpoint, according to decreasing values of  $v_i$ .

3) Evaluate the objective (9) for the current ordering by successively including users from the top of the order.

4) Pick the best objective over all the intervals and user sets examined.

The complexity of the above algorithm for  $N$  users is  $O(N^3 \log N)$ , and is guaranteed to give the optimal solution.

### III. OPTIMAL UPLINK SCHEDULING

In this part, we attempt to provide a greedy, low-complexity, approximate solution to the convex maximization discussed before.

#### A. QRP Algorithm

Recall that the sorting measure used in the optimal solution for fixed  $L$  was of the form

$$v_i = Q_i g_i(a_i) / P_i$$

which suggests a greedy algorithm that ranks users by the same measure without  $L$ . We hence propose the following simple scheduling scheme that may be more suitable for practical implementation:

#### QRP algorithm:

1) Sort users in decreasing order of the measure  $v_i = Q_i R_i^0 / P_i$

assuming no interference from other users while computing  $R_i^0$ .

2) Add user  $i$  in order starting from the top of the list while maintaining and updating the

value of  $O = \sum_{j < i} Q_j R_j$ , where  $R_j$  now takes into account interference from all added users.

3) Stop if adding the next user reduces  $O$ , and allow transmission of all added users at their peak powers and rates as computed.

As we will see in our simulations section, this simple algorithm captures most of the benefits of optimal uplink scheduling, and has the properties alluded to in the introduction. In other words, the chosen user sets from the above algorithm tend to be one of the following types:

- A single "strong" user with high  $P_i$
- A group of "weak" users with low  $P_i$ , and often high  $Q_i$ .

This observation is consistent with the common intuition relating to the nature of interference in CDMA systems.

#### B. Other Sub-optimal Algorithms and Benchmarks

One benchmark algorithm is the optimal algorithm given in previous sections. It gives the best possible performance. Another benchmark algorithm is the MaxQR algorithm. It serves the user (one-at-a-time) with the maximum queue length and data rate product,  $\text{argmax } Q_i R_i$ . This algorithm serves as a lower bound. We will compare with this algorithm to evaluate the gains of different sub-optimal algorithms. Round Robin and fully simultaneous transmission are considered too far from optimal and perform very poorly in most of the cases, and are thus ignored here.

We will provide some other sub-optimal algorithms, which perform less well than the above proposed QRP algorithm. However, they offer simplicity in implementation by using less processing and signaling power. One example of such algorithm is the *Average-SIR algorithm*. Calculate average SIR,  $g_{\text{avg}}$  among all the users (with non-empty queue). There are two ways to compute users SIR at this stage.

A1. Assume users (with non-empty queue) transmit one-at-a-time.

A2. Assume users (with non-empty queue) transmit simultaneously.

#### IV. SIMULATION RESULTS

1) The location of the mobiles is assumed to be uniformly distributed in the cell area.

2) It is assumed that the link gains have the following form  $G_i(k) = d_i^{-4}(k)A_i(k)B_i(k)$  where  $d_i(k)$  is the distance from the  $i$ -th mobile to the base station at time instant  $k$ ,  $A_i$  is a log-normal distributed stochastic process (shadowing).  $B_i$  is a fast fading factor (Rayleigh distributed).

3) It is assumed that the cell diameter is 3 km.  $d_i(k)$  is a 2-D uniformly distributed random variable.

4) It is assumed that the standard deviation of  $A_i$  is 7 dB.

5) It is assumed that all users share 1.25MHz bandwidth.

6) It is assumed that packet length is exponentially distributed

7) Simulation time = 20 minutes.

The total average queue length of all users is shown in Figure 1. Note that QRP scheduling has more than 50% gain over Max QR algorithm. Furthermore, the queue length of a typical "weak" mobile user (see Figure 2(b)), and the queue length of a typical "strong" mobile user (see Figure 2(a)), clearly indicate that QRP algorithm improves fairness among mobile users as well as increases throughput.

Figure 2(a), clearly indicate that QRP algorithm improves fairness among mobile users as well as increases throughput. It shows that QRP algorithm, which allowing either multiple "weak" mobile users transmitting simultaneously or a single "strong" mobile user transmitting, achieve most performance gain by reducing the queue lengths of the "weak" mobile users.

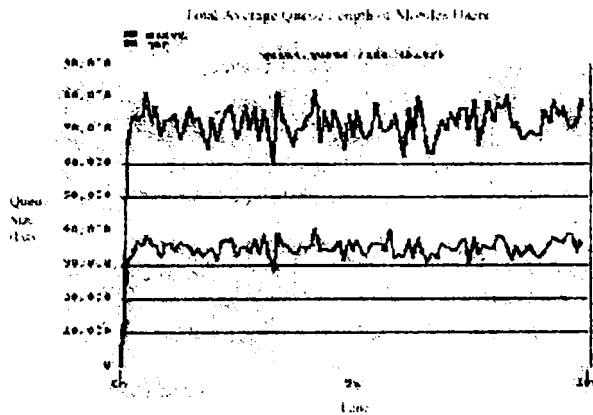
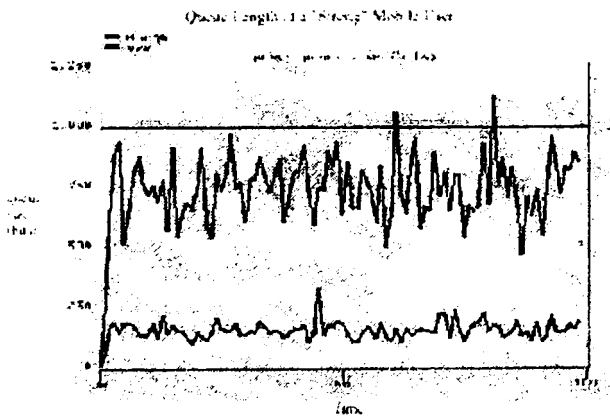
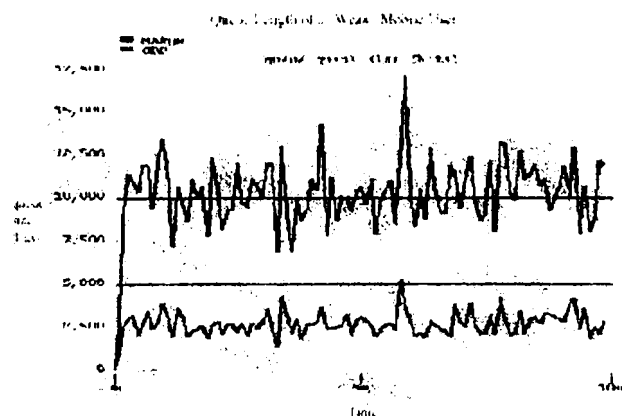


Fig. 1. Total average queue length



a)



b)

Fig. 2. Queue length of (a) a typical "strong" mobile user and (b) a typical "weak" mobile

We have proposed optimal uplink scheduling algorithms for a single CDMA cell, and related efficient approximate algorithms for practical implementation. Simulations demonstrate substantial performance improvement with these algorithms, and we also discuss the implementation requirements they entail. Further research is required to address multi-cell systems, and also to incorporate the effects of soft-handoff, which may have a significant influence in uplink scheduling.

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