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Noise impulse generation with convenient characteristics in time and frequency domain

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Abstract - In the paper "Impulse generation with appropriate amplitude, length, inter-arrival, and spectral characteristics," by I.Mann *et.al.*, [1] the authors present what the guest editors believe is the most recent statistical model of nonstationary impulse noise.

The Henkel/Kessler (HK) model discussed in that paper proved to be a good fit for all measured impulse noise voltage amplitude distributions collected in both the networks of Deutsche Telekom (DT) and British Telecom (BT). Nevertheless, in order to facilitate the use of the results of Tough and Ward [3] on random noise generation with prescribed amplitude and spectral characteristics, a Weibull type density was investigated as a possible alternative since it simplfies an approximate realization of the stochastically varying spectral properties. The authors recognized that HK model is a better fit than the Weibull density and can be considerd as more realistic while suggesting that in further studies the Tough- Ward method for the DT data sets will be finalized.

This paper proposes a suitable method for simulating impulse noise with Henkel/Kessler amplitude probability density function and impulse length according a stable probability density function.

***Keywords*- impulse noise, nonstationary noise, xDSL, α-stable distribution.**

I. INTRODUCTION

Telecommunication companies and equipment manufacturers are interested in modeling the impulse noise that is disturbing the xDSL systems. I. Mann *et al.* [1] present a statistical model considered to be the most recent nonstationary noise impulse model. A method for the simulation of noise impulses with given amplitude, length and spectral density characteristics is proposed.

Impulse noise is considered to be one of the main causes of signals' degradation in xDSL systems. That is why companies are interested in modeling this noise. A noise impulse model must describe, in a statistical sense, both time domain and frequency domain impulses' properties.

In Mann & Henkel model, the parameters are chosen according to the empirically obtained statistics when measurements in the British Telecom and Deutsche Telekom networks were performed. The authors abandoned the Henkel-Kesler (HK) model which was proven to be more realistic for the modeling practice in favour of Weibull distribution in order to facilitate the use of the results of Tough and Ward [3] on random noise generation with prescribed amplitude and spectral characteristics. In the following, we shall apply the Tough and Ward method to the HK model.

II. STATISTIC MODELING FOR THE PROBABILITY DENSITY OF IMPULSES AMPLITUDE

The original model known as HK (Henkel- Kessler) model was proposed in [2]

Probability density function of the impulses' amplitudes is given by:

$$f_1(u) = \frac{1}{240u_0} e^{-\left(\frac{u}{u_0}\right)^{1/5}}, u_0>0 \quad (1)$$

It tells that this is a probability density symmetric to the origin.

It can be demonstrated that the transformation of variable $y = \left(\frac{u}{u_0}\right)^{\frac{1}{5}}$ leads to a gamma symmetric distribution $\gamma = \frac{1}{2\Gamma(5)} y^4 e^{-|y|}$.

The model was proven to be appropriate for the information gathered from both networks (BT and DT). Nevertheless, in order to facilitate the use of the results of Tough and Ward on noise generation, a symmetric Weibull probability density was used

$$P(y) = \tfrac{1}{2}\alpha b|y|^{\alpha-1} e^{-b|y|^{\alpha}} \quad (2)$$

The parameters for theWeibull and HK models in BT and DT network measurements are given in table 1:

Table 1

	Weibull		HK
	a	b	u_0
BT(CP)	0,263	4,77	9,12 μV
DT(CP)	0,486	44,4	23,23 nV
DT(CO)	0,216	12,47	30,67 nV

CP – Customer Premises
CO- Central Office

In order to test the xDSL systems, synthetic impulses are generated using the Tough-Ward method that combines the amplitude probability density function with the correlation function model to produce impulses with appropriate time domain and frequency domain properties.

First of all, this method assumes to find a memoryless nonlinear transform (MNLT) that maps between a zero-mean, unit variance Gaussian probability density and the required probability density function. This is then used to calculate the relationship between correlation coefficients of the two processes. Once this relationship is found, then it is possible to impose a correlation onto the input Gaussian sequence of given length by filtering with a FIR having a spectrum that corresponds to the input correlation function.

The Gaussian filtered sequence is fed to the memoryless nonlinear transform in order to generate impulse with given amplitude and spectral density characteristics.

To find the memoryless nonlinear transform $y = g(x)$, the cumulative distribution functions for the normal pdf and for the required pdf are equated

$$\int_y^{\infty} \frac{1}{240u_0} e^{-(\frac{u}{u_0})^{1/5}} du = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-\frac{t^2}{2}} dt \qquad (4)$$

Left side integral can be calculated:

$$I_l = \frac{1}{48}\left(v^4 + 4v^3 + 12v^2 + 24v + 24\right)e^{-v} \qquad (5)$$

where $v = \left(\frac{y}{u_0}\right)^{1/5}$

For the purpose of this paper it is recommended to use the incomplete gamma function given by:

$$P(x,a) = \frac{1}{\Gamma(a)} \int_0^x t^{a-1} e^{-t} dt, x \in R, a \in R_+ \qquad (6)$$

Starting from (6) it is easy to obtain the left side integral:

$$I_l = -\frac{1}{2} P((\frac{y}{u_0})^{1/5}, 5) + \frac{1}{2} \qquad (7)$$

The right side integral is known

$$I_r = -\frac{1}{2} erf(\frac{x}{\sqrt{2}}) + \frac{1}{2} \qquad (8)$$

where $erf(u) = \frac{2}{\sqrt{\pi}} \int_0^u e^{-t^2} dt$ is the error function.

The memoryless nonlinear transform (MNLT) $y = g(x)$ can be numerically obtained in Matlab from the incomplete gamma function *gammainc(y,a)* and the reverse of the error function *erfinv(w)*. The result is a symmetric with respect to the origin function $y = g(x)$ (Fig 1 left)

In the following the correlation function of the process y is evaluated. This can be expressed in the form:

$$\langle \eta(0), \eta(t) \rangle = \frac{1}{2\pi} \sum_{n=0}^{n=\infty} \frac{R_G(t)}{2^n n!} \cdot \left[\int_{-\infty}^{\infty} \exp(\frac{x^2}{2}) H_n\left(\frac{x}{\sqrt{2}}\right) g(x) dx \right]^2 \qquad (9)$$

where H_n are the Hermite polynomials of nth degree. Once we have evaluated the integrals

$$\int_{-\infty}^{\infty} \exp(\frac{x^2}{2}) H_n\left(\frac{x}{\sqrt{2}}\right) g(x) dx \qquad (10)$$

we have a power series representation of the mapping between the correlation functions of the input Gaussian and the output non-Gaussian processes. This series are rapidly convergent.

The integral (10) is numerically evaluated and the resulting polynomial is used to generate a lookup table relating the input and output correlation coefficients.

So far, we have established a readily evaluable and invertible mapping between the correlation functions of the input Gaussian and the output non-Gaussian processes related by the nonlinear transformation $y = g(x)$.

Using this we can tailor the correlation properties of the input Gaussian process through the methods described in [1].

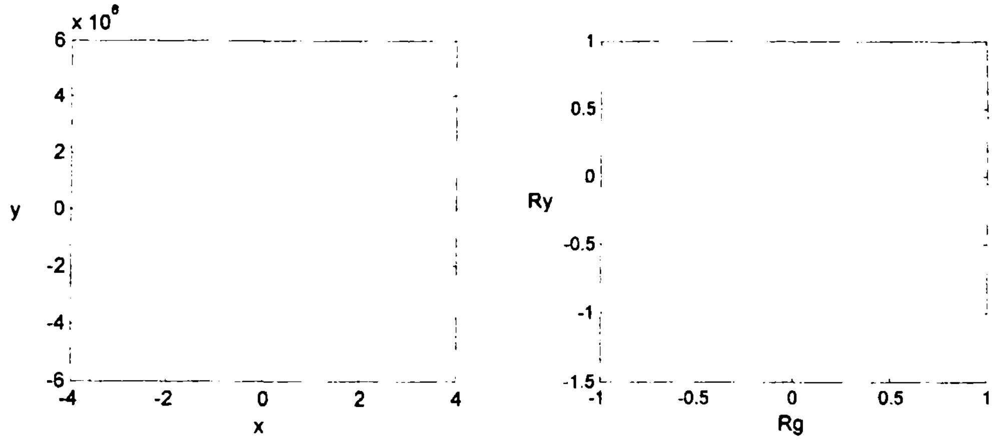


Fig. 1. The mapping between the input and the output correlation functions under the MNTL for HK density

III. A NEW MODEL FOR LENGTH DISTRIBUTION

The approach for modelling the probability density of impulse duration in [2] is left unchanged to be a sum of two log-normal forms.

$$f_l(t) = B\frac{1}{\sqrt{2\pi}s_1 t}e^{-\frac{1}{2s_1^2}\ln^2\left(\frac{t}{t_1}\right)} + (1-B)\frac{1}{\sqrt{2\pi}s_2 t}e^{-\frac{1}{2s_2^2}\ln^2\left(\frac{t}{t_2}\right)} \qquad (11)$$

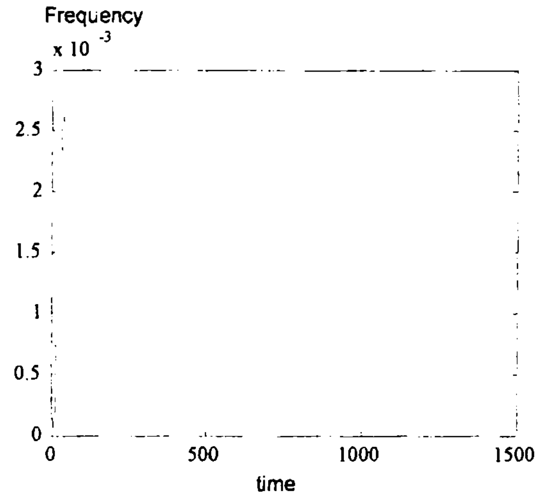
The typical parameters of the model are given in table 2

Table 2

	B	S_1	T_1 (µs)	S_2	T_2 (µs)
BT(CP)	0,45	1,25	1,3	21,5	129
DT(CP)	1	1,15	18	-	-
DT(CO)	0,25	0,75	8	1,0	125

In this paper we propose an alternative model for length probability density function, a stable distribution

$$p_b(x) = \begin{cases} 0 & x < 0 \\ \frac{b}{\sqrt{2x}}e^{\frac{-b^2}{2x}}x^{-\frac{3}{2}} & x > 0 \end{cases}$$



The stable law is a direct generalization of Gaussian distribution and in fact includes the Gaussian as a limiting case.

The main difference between the non-Gaussian stable distribution and the Gaussian distribution is that the tails of the stable density function decay less rapidly than the Gaussian density function. This characteristic of the stable distribution is one of the main reasons why the stable distribution is suitable for modeling signals and noise of impulsive nature.

The stable distribution is very flexible as a modeling tool in that it is determined by four parameters: 1) the location parameter a 2) the scale parameter b, also called dispersion, 3) the index of skewness β and 4) the characteristic exponent α. For more information about the stable distribution, we refer the reader to appendix.

IV. CONCLUSIONS

We presented the Tough-Ward procedure in the case of Henkel Kessler model for the amplitude probability density function, an unsolved problem in June 2002 when the paper [1] was published. A new model for length distribution is proposed. This is an α-stable distribution with $\alpha = 0.5$.

APPENDIX

STABLE DISTRIBUTIONS

A distribution function (d. f.) is said to be stable if for every $n \geq 2$ there exist constants a_n and $b_n > 0$ such that

$$F(x) = F^{*n}(b_n x + a_n). \tag{A.1}$$

F^{*n} is the n-fold convolution of F with itself.
The corresponding characteristic function (c.f.) $f := E[e^{itX}]$ is also called stable. A d.f. F is stable if and only if (iff) for every collection $X_0, X_1, \dots, X_n$ of $n+1$ mutually independent random variables with a common distribution (i.i.d r.v.), $X_k \sim F$, $0 < k \leq n$, $n \geq 2$, there exist constants a_n and $b_n>0$ such that

$$X_0 \sim \frac{1}{b_n}(X_1 + \cdots + X_n - a_n) \tag{A.2}$$

Every stable d. f. belongs to its own domain of attraction [4].

Proposition 1 [4], [5] Only the norming constants $b_n = n^{1/\alpha}$ are possible.

Proposition 2 [5] A non-degenerate d.f. F with *c. f.* f is stable iff there exit real constants α, β, a and b with $0 < \alpha \leq 2$, $|\beta| \leq 1$ and $b>0$, such that

$$\ln f(t) = iat - b|t|^{\alpha}[1 + i\beta \operatorname{sgn} t.\omega_{\alpha}(t)] \tag{A.3}$$

where

$$\omega_{\alpha}(t) = \begin{cases} \tan\frac{\pi\alpha}{2} & \text{if } \alpha \neq 1 \\ \frac{2}{\pi}\ln|t| & \text{if } \alpha = 1 \end{cases}$$

The parameter α is the characteristic exponent and β is the skewness parameter. The parameters a and b in (A.3) are respectively location and scale parameters. We will denote a stable *d. f.* F by $F = S(\alpha, \beta; a, b)$.

Example (i) For $F = S(1,0;0,1)$ we obtain $f(t) = e^{-|t|}$ so that F is the Cauchy *d. f.*

(ii) For $F = S(1/2, -1; 0, b)$ we have

$$\ln f_b(t) = -b\sqrt{|t|}[1 - i \operatorname{sgn} t] \tag{A.4}$$

and F has the Pearson density

$$p_b(x) = \begin{cases} 0 & x < 0 \\ \frac{b}{\sqrt{2x}} e^{\frac{-b^2}{2x}} x^{-\frac{3}{2}} & x > 0 \end{cases} \tag{A.5}$$

and

$$F_b(x) = 2(1 - N(\frac{b}{\sqrt{x}})), \qquad x>0 \tag{A.6}$$

where

$$N(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-y^2/2} dy \tag{A.7}$$

(iii) If $\alpha = 2$ then f is normal.
Using (A.3)) one obtains
Proposition 3 Let $F_k = S(\alpha_k, \beta_k; a_k, b_k)$, k=1,2, be stable such that $\alpha_1 = \alpha_2 = \alpha$. Then $F = F_1 * F_2$ is also stable and we have $F = S(\alpha, (\beta_1 b_1 + b_2\beta_2)/(b_1 + b_2); a_1 + a_2, b_1 + b)$

Proposition 4 Let $F = S(\alpha, \beta; a, b)$ be stable. Then the following assertions are true

(i) $F_1(x) := F(\mu x + \nu)$, $\mu>0$ is also stable. We have $F = S(\alpha, \beta; a_1, b_1)$ where $b_1 = b/\mu^{\alpha}$ and

$$a_1 = \begin{cases} \frac{a - \nu}{\mu} & \text{if } \alpha \neq 1 \\ \frac{a - \nu - (2/\pi)\beta \ln \mu}{\mu} & \text{if } \alpha = 1 \end{cases} \tag{A.8}$$

(ii) F is symetric with respect to x_0 iff $a = x_0$ and either $\alpha = 2$ or $\beta = 0$

(iii) F is one-sided *d.f.* iff $\alpha < 1$ and $|\beta| = 1$. In this case a=sup$\{x : F(x) = 0\}$ if $\beta = -1$ and a=inf$\{x : F(x) = 1\}$ if $\beta = 1$

(iv) If $-1 \leq \beta < 1$ then the probability density function $p(x, \alpha, \beta, a, b)$ decreases as const. $x^{-1-\alpha}$ for $x \to \infty$. If $\beta = 1$ and $\alpha < 1$ for $x \to a - 0$, and if $\beta = 1$, $\alpha \geq 1$ for $x \to \infty$, $p(x, \alpha, \beta, a, b)$ decreases exponentially.

(v) $E|X|^r < \infty$ if $0 \leq r < \alpha$ If $\alpha = 2$ the stable distribution is Gaussian and $E|X|^r < \infty$ for all $r \geq 0$.

Thus, if $\beta < 1$ stable laws have inverse power (i.e. algebric) tails. This proves that the tails of stable laws are much thiker than those of the Gaussian distribution. An important consequence of (*iv*) is the nonexistence of the second order moment (except for the case $\alpha = 2$).

Proposition 5 Let X be an α -stable random variable. If $0 < \alpha < 2$, then

$$E|X|^p = \infty \quad \text{if } p \geq \alpha$$

and

$$E|X|^p < \infty, \text{ if } 0 \leq p < \alpha.$$

If $\alpha = 2$, then

$$E|X|^p < \infty \text{ for all } p \geq 0.$$

All non-Gaussian stable distributions have infinite variance.
Let $X_1, ..., X_n$ be a collection of i.i.d. r.v. and $X_{(n)}$ the largest among them. If the X_j have the stable density (A.4) then

$$P\{n^{-2} X_{(n)} \leq x\} \to e^{-b\sqrt{2/\pi x}} \qquad \text{(A.9)}$$

Proof: If a limit distribution G exists we have $F^n(n^2 x) \to G(x)$ at all points of continuity. Passing to logarithms we get

$$n[1 - F(n^2 x)] \to -\log G(x) \qquad \text{(A.10)}$$

We have for $n \to \infty$

$$n\left[2N\left(\frac{b}{\sqrt{xn^2}}\right) - 1\right] \to \frac{b\sqrt{2}}{\sqrt{\pi x}} = -\log G(x)$$

(A.11)

Propositon 6 For fixed $0 < \alpha < 1$ the function $\gamma_\alpha(s) = e^{-s^\alpha}$ is the Laplace transform of a distribution G_α with the following properties:
G_α is stable

$$x^\alpha [1 - G_\alpha(x)] \to \frac{1}{\Gamma(1-\alpha)} \quad x \to \infty \qquad \text{(A.12)}$$

$$e^{x^{-\alpha}} G_\alpha(x) \xrightarrow{x \to 0} 0. \qquad \text{(A.13)}$$

Proposition 7 Suppose that F is a d.f. concentrated on $(0, \infty)$ such that

$$F^{*n}(a_n x) \to G(x) \qquad \text{(A.14)}$$

(at points of continuity) where G is a proper distribution not concentrated at a single point. Then:

(i) There exists a function L that varies slowly at infinity and a constant α with $0 < \alpha < 1$ such that

(ii)

$$1 - F(x) \approx \frac{x^{-\alpha} L(x)}{\Gamma(1-\alpha)} \qquad \text{(A.15)}$$

(iii) Conversely if F is of the form (A.15) it is possible to choose a_n such that

$$\frac{nL(a_n)}{a_n^\alpha} \to 1 \qquad \text{(A.16)}$$

and in this case (A.14) holds with $G = G_\alpha$

(from the theory of regular variation a positive function L defined on $(0, \infty)$ varies slowly (at ∞) if for all $x > 0$, $\frac{L(sx)}{L(s)} \xrightarrow{s \to \infty} 1$).

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