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Parameter estimation of the chirp signal

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Abstract – The paper presents the characteristics of the parameters of the chirp signal, and the situations where this signal intervene. Using Kalman filtering, a few parameters of the chirp signal will be estimated, such as frequency and phase. The estimation results and the impact of changes in the Kalman Filter's are analysed graphically, and finally some conclusions are drawn.

Keywords: Kalman Filter, chirp signal, simulation, estimation

I. INTRODUCTION

The solution of the optimal filter problem is a filter weighting function that tells us how the past values of the input should be weighted in order to determine the present value of the output, the optimal estimate. The Kalman solution has two main features: ones a vector modeling of the random processes and a recursive processing of the noisy measurement data.

The input data makes part of the common case of noisy sensor measurements. The time-varying ratio of the pure signal to the electrical noise affects the quantity and the quality of the information. The result is that the measured information must be qualified as it is interpreted as part of an overall sequence of estimates.

The main feature of Kalman filtering is the recursive operation mode. The key element in any recursive procedure is the use of the results of the previous step to aid in obtaining the desired result for the current step.

II. DISCRET-TIME MODEL

Discrete-time processes may arise in two ways: the situation where a sequence of events takes place naturally in discrete steps, with a fixed or random variable for each step length. Another solution is to sample a continuous process at discrete time. Irrespective of how the discretization arises, the general format is:

$$x_{k+1} = \phi_k x_k + w_k \quad (1)$$

$$y_k = B_k x_k \quad (2)$$

where

x_k – vector state of the process at time t_k, $x_k = x(t_k)$

ϕ_k - matrix that relates x_k to x_{k+1}

w_k - vector whose elements are white sequences

B_k – linear connection matrix between output y_k and state x_k

Consider a dynamic process described by an n-th order difference equation of the form:

$$y_{i+1} = a_0 \cdot y_i + \ldots + a_{n-1} \cdot y_{i-n-1} + u_i, i \geq 0 \quad (3)$$

This difference equation can be re-written as:

$$x_{i+1} = \begin{bmatrix} y_{i-1} \\ y_i \\ y_{y-1} \\ \\ y_{i-n+2} \end{bmatrix} = \quad (4)$$

$$= \begin{bmatrix} a_0 & a_1 & a_{n-1} \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ \vdots & \vdots & \vdots \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} y_i \\ y_i - 1 \\ y_{y-2} \\ \\ y_{i-n-1} \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \\ .. \\ 1 \end{bmatrix} u_i$$

which leads to the state space model form from equations (1) and (2), with $B_k = [1\ 0\ \ldots\ 0]$.

Generally, a continuous process can be described by

$$x = Fx + Gu \quad (5)$$

where u is a vector forcing function whose elements are white noise.

One special case of evaluating ϕ_k is in case of fixed parameters for the dynamical system (i.e., F is a constant), the state transition matrix (STM) may be written as an exponential series:

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$$\phi_k = e^{F\Delta t} = 1 + (F\Delta t) + \frac{(F\Delta t)^2}{2!} + \dots \quad (6)$$

where Δt is the step size.

III. KALMAN FILTER

The Kalman filter is essentially a set of mathematical equations that implements a predictor-corrector type estimator that minimizes the estimated error covariance.

The random process that has to be estimated can be modeled in form of equation (1). The process measurement, at discrete points is:

$$z_k = H_k x_k + v_k \quad (7)$$

where

z_k – vector measurement at time t_k

H_k – matrix given the ideal connection between the measurement and the state vector at time t_k

v_k – measurement error

The covariance matrices for the vectors v_k and w_k are given by:

$$E\left[w_k w_i^T\right] = Q_k \quad (8)$$

$$E\left[v_k v_i^T\right] = R_k \quad (9)$$

The estimation error is defined as the difference between the state and his a priori estimate (' best estimate):

$$e_k^- = x_k - \hat{x}_k^- \quad (10)$$

With the assumption of a prior estimate $\hat{x}_k^-$ the measurement z_k can be used to improve a prior estimate:

$$\hat{x}_k = \hat{x}_k^- + K_k\left(K_k - H_k \hat{x}_k^-\right) \quad (11)$$

where $\hat{x}_k$ is the update estimate and K_k an blending factor. The expression for the error covariance matrix associated with the update (a posteriori) estimate is:

$$P_k = E\left[e_k e_k^T\right] = E\left[\left(x_k - \hat{x}_k\right)\left(x_k - \hat{x}_k\right)^T\right] \quad (12)$$

The expressions (7) and (11) will be substituted in (12). The next step is to minimize the expression of P, the sum of the mean-square error (12), differentiating P with respect to K. This particularly K_k solution, after setting the derived equal to zero, is called the Kalman gain:

$$K_k = P_k^- H_k^T\left(H_k P_k^- H_k^T + R_k\right)^{-1} \quad (13)$$

The covariance matrix, associated with the optimal estimate, is now:

$$P_k = (I - K_k H_k)P_k^- \quad (14)$$

The update estimates $\hat{x}_k$ can be projected ahead via the transition matrix:

$$\hat{x}_{k+1}^- = \phi_k \hat{x}_k \quad (15)$$

The error covariance matrix associated with the updated estimated has the expression:

$$P_{k+1}^- = E\left[e^-_{k+1} e^-_{k+1}\right] = \phi_k P_k \phi_k^T + Q_k \quad (16)$$

The table 1 offers a complete picture of the Kalman filter operation that processes discrete measurements (input) into optimal estimates (the output).

Table 1

Predict (time update)	Correct (measurement update)
(1) Project the state ahead $\hat{x}_{k+1}^- = \phi_k \hat{x}_k$ (2) Project the error covariance ahead $P_{k+1}^- = \phi_k P_k \phi_k^T + Q_k$	(1) Compute the Kalman gain $K_k = P_k^- H_k^T\left(H_k P_k^- H_k^T + R_k\right)^{-1}$ (2) Update estimate with measurement z_k $\hat{x}_k = \hat{x}_k^- + K_k\left(K_k - H_k \hat{x}_k^-\right)$ (3) Update the covariance error $P_k = (I - K_k H_k)P_k^-$

IV CHIRP SIGNAL

Chirp signals are encountered in many different engineering applications including radar, active sonar and passive sonar systems. The main characteristic of the chirp signals is the linear change of their instantaneous frequencies, and therefore they have often been used in representing signals with time varying spectra. Parameter estimation of chirp signals has been of great interest in the past, and a wide variety of estimation procedures have been proposed and studied.

The chirp signal can be formed in two sweep modes. A unidirectional one, Fig. 1, where the cosine frequency is immediately reset to f(0), the initial frequency, after the sweep period is traversed. When the sweep mode is bi-directional, Fig. 2, the frequency sweep reverses direction half way through the period, and returns to f(0) along a symmetrical trajectory.

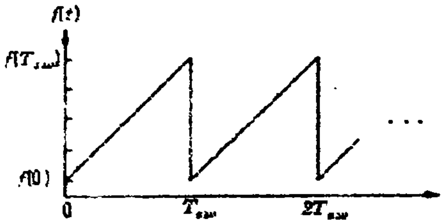

Fig.1 Chirp signal with unidirectional sweep

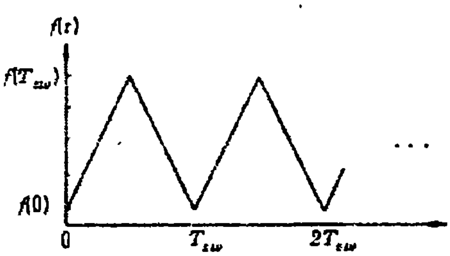

Fig.2 Chirp signal with bi-directional sweep

The frequency sweep of a chirp signal can be cosine, similar to linear, quadratic, or logarithmic. The linear frequency sweep uses an instantaneous frequency sweep f(t), $f(t)=f(0)+\beta t$, where $\beta=\frac{f(t_g)-f(0)}{t_g}$, t_g – target time, $f(t_g)$ – target frequency. Quadratic frequency sweep uses an instantaneous frequency sweep f(t) of $f(t)=f(0)+\beta t^2$ and the logarithmic once with the frequency sweep $f(t)=f(0)+10^{\beta t}$, where $\beta=\frac{\log[f(t_g)-f(0)]}{t_g}$.

V. INSTANTANEOUS FREQUENCY AND PHASE

The chirp signal to be dealt with is a linear, quadratic one, given by

$$\beta=(f_1-f_0)t_1^{-p} \tag{17}$$

$$y=\cos\left(2\pi\frac{\beta}{1+p}t^{1+p}+f_0 t+phi/360\right) \tag{18}$$

where p is the polynomial order and *phi* de initial phase.
The model of the signal that is here considered can be expressed as:

$$y(t)=A\cos\phi(t) \tag{19}$$

with A constant. The instantaneous frequency, $f_i(t)$, of the signal is

$$f_i(t)=\frac{1}{2\pi}\frac{d\phi(t)}{dt} \tag{20}$$

VI. SIMULATIONS MODEL USING KALMAN FILTERING

We consider the problem of estimating the parameters of a chirp signal observed in additive noise. This paper presents the results of the computer simulation for a signal with constant amplitude and linear frequency modulation defined through the equations (13) and (14). The considered values are the instantaneous frequency at time 0, f_0=50 Hz, instantaneous frequency f_1 achieved at time t_1=1s is f_1 =500Hz, p=1. The signal will be discretized in 21 points. The first plot, figure 3, shows the 'real' phase, the noise affected phase and the estimated one. The Kalman filter parameters for this estimation are: f_i =0.98, H=1, Q=1, R=1, X=0, P=1, I=1. The implemented algorithm follows the steps presented in Table 1.

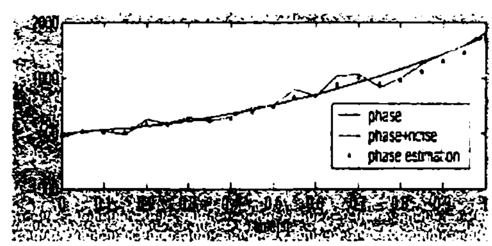

Fig.3 Phase estimation

After the first eight steps, the filter settles down to a steady – state condition where the Kalman filter gain is about 0.8274. Fig. 4 shows the evolution of the phase estimation corresponding to the first part of the Kalman algorithm.

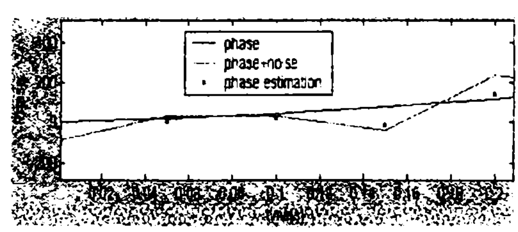

Fig. 4 First part of the phase estimation

The expression of the instantaneous frequency (17) was computed based on equation (16).

$$F=\frac{2\beta}{1+p}\cdot t+\frac{f_0}{2\pi} \tag{21}$$

Using the same Kalman filter parameters like those in the phase estimation, but for a greater number of points, 101, the simulation for the frequency parameter estimation is shown in Fig. 5.

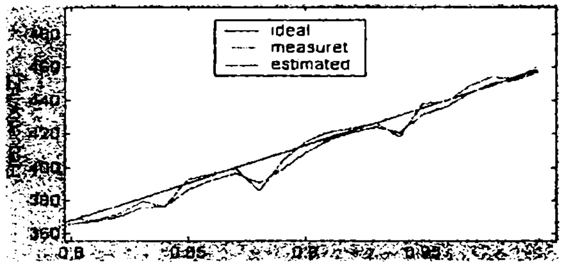

Fig.5 Frequency estimation

After the first ten steps the Kalman filter stabilizes by 0.618. If we repeat the simulation for other filter parameters like the state transition matrix (scalar in this case) ϕ_k, we can observe a different behavior of the estimated result. For ϕ_k=1.45, the phase estimation will have the following allure, Fig 6.

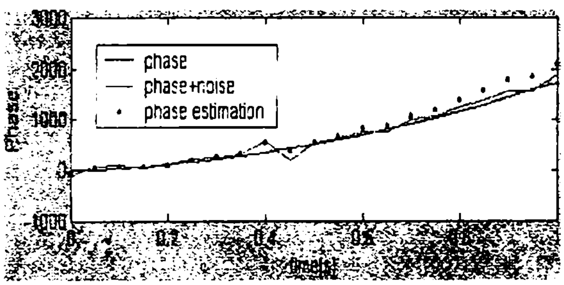

Fig.6 Phase estimation with $\phi_k > 1$

If we choose a sub unitary value for the transition matrix ϕ_k and do not change the time at which the noisy measurements of this process are taken. Δt, the resulted frequency estimation is shown in Fig. 7.

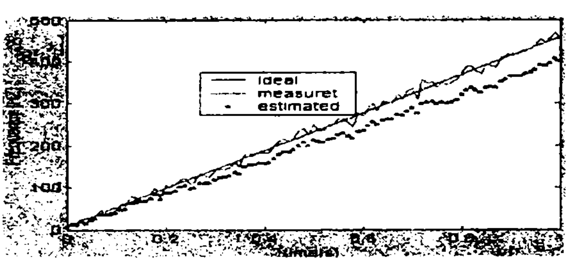

Fig.7 Frequency estimation with $\phi_k < 1$

CONLUSIONS

The plot for the estimation of the phase of the analyzed chirp signal gives us a parabola, a second-degree function. We observe that, as time goes by, the filter depends more on the measurements and less on the initial assumptions. The estimated frequency has a linear evolution, but also, the simulation result also shows that the filter parameters need to be adapted for an optimal result. If this doesn't happen, then the estimation will follow too much the measurement and the noise that affects it.

The importance of the correct determination of the filter parameters ϕ_k – state trans t on matrix, H_k – measurement relationship to x, the noise sequence Q_k and the measurement error R_k, have been evidenced in the plots form Fig. 6 and 7. The estimation, after a higher number of steps, doesn't settle down to an optimum estimation but increases or decreases from the actual process x(t).

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