Buletinul Ştiinţific al Universităţii "Politehnica" din Timişoara

Seria ELECTRONICĂ şi TELECOMUNICAŢII
TRANSACTIONS on ELECTRONICS and COMMUNICATIONS

Tom 49(63), Fascicola 1, 2004

State-Space Control Structures for Buck Converters with/without Input Filter

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Abstract – Power supplies normally provide a constant output voltage. In most of the applications a DC/DC-converter is controlled by a voltage mode or a current mode controller. Often these controller types are combined with feed forward techniques of the input voltage or output current, because these variables are disturbance variables. In this a er three state-s ace control structures are introduced. Based on the state-space representation an easy controller is designed first. An improved one works with an inner one-cycle controller. The third controller is designed for a buck converter with a minimized input filter.

Keywords: state-space controller, buck converter, input filter

I. INTRODUCTION

Power supplies normally provide a constant output voltage. In most of the applications a DC/DC-converter is controlled by a voltage mode or a current mode controller [1]. These controller types are well known and there are a lot of PWM controller ICs on the market. The controllers are often combined with feed forward techniques of the input voltage or output current, because these variables are disturbance variables. There are different types of current mode control like peak current mode or average current mode control, a kind of cascade control. Furthermore there are some other controllers types like sliding mode control, one-cycle control, optimal time control, two-step control or delta sigma control.

In this paper three state-space control structures are introduced. Based on the state-space representation an easy controller is designed first. An improved one works with an inner one-cycle controller and an outer I-controller for improving robustness. The third controller is designed for a buck converter with an input filter. The advantages of state-space controllers are pole placement and easy implementation. The input filter requires no resistor for damping and the filter elements can be very small. A disadvantage is low robustness.

II. MODELING A BUCK CONVERTER

Modeling a converter using state-space averaging is well known since many years [2]. The circuit is shown in fig. 1.

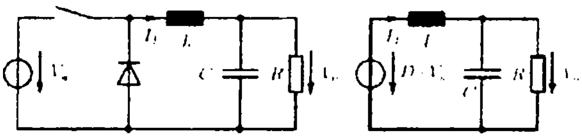

Fig1: Circuit of the buck converter and low frequency equivalent circuit

The state-space description for the continuous conduction mode is given with (d is the duty cycle, index 1: switch conducting, index 2: diode conducting)

$$\dot{x} = A_1 \cdot x + b_1 \cdot u \quad \textit{for} \quad 0 \le t \le d \cdot T,$$
$$\dot{x} = A_2 \cdot x + b_2 \cdot u \quad \textit{for} \quad d \cdot T \le t \le T.$$

The idea of state-space averaging is joining the two equations together considering the action time of each equation. We will get

$$\dot{x} = A \cdot x + b \cdot u$$

with

$$A = A_1 \cdot d + A_2 \cdot (1-d) \text{ and } b = b_1 \cdot d + b_2 \cdot (1-d).$$

Every signal can be represented by a DC part and an AC part (e. g. $h = H + \tilde{h}$). Than the small signal control-to-output transfer function can be calculated by [3]

$$\frac{\tilde{v}_o}{\tilde{d}} = c \cdot (s \cdot I - A)^{-1} [(A_1 - A_2) \cdot X + (b_1 - b_2) \cdot V_g] + (c_1 - c_2) \cdot X \,,$$

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where X is the steady state vector $X = -A^{-1} \cdot b \cdot V_g$. Therefore a buck converter has the control-to-output transfer function

$$\frac{\tilde{v}_o}{\tilde{d}} = \frac{V_g}{1 + \underline{s}\frac{L}{R} + \underline{s}^2 LC}.$$

III. STATE-SPACE CONTROL

The block diagram of a state-space controller is shown in fig. 2 [4].

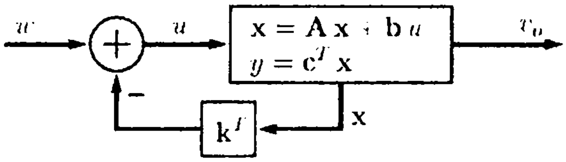

Fig 2: Block diagram of a state-space controller

In fig. 2 the manipulated variable becomes $u = w - k^T \cdot x$. This requires that all state variables are measurable. The state equation than is

$$x = Ax + bu = Ax + b(w - k^T x) = (A - bk^T)x + bw.$$

The system matrix is now $A - bk^T$. Therefore the closed loop poles are the zeros of the characteristic polynomial

$$P(\underline{s}) = \det(\underline{s}I - A + bk^T).$$

It can be shown that the controllability matrix of the system matrix A and the input vector b of the buck converter has full rank [5]. So the buck converter is fully controllable by a state-space controller.

But in fact state-space controllers are proportional controller. So there will be a steady state error, if the output current or the input voltage is varying. To minimize the error the system description can be rewritten in a way where the state variables are replaced by the errors of the original state variables. We get now

$$e = \begin{bmatrix} e_L \\ e_C \end{bmatrix} = \begin{bmatrix} i_L - \frac{V_o}{R} \\ v_o - V_o \end{bmatrix}.$$

Furthermore is $\dot{e} = \dot{x}$. The state-space equations becomes

$$\dot{e} = \begin{bmatrix} 0 & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{RC} \end{bmatrix} e + \begin{bmatrix} \frac{D}{L} & -\frac{1}{L} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_g \\ V_o \end{bmatrix}.$$

The duty cycle consists a DC part and an AC part $d = D + \tilde{d}$. The DC part D correspondents with feed forward of the input voltage. So the AC part $\tilde{d}$ is the manipulated variable.

$$d = D + \tilde{d} = \frac{V_o}{V_g} + \frac{1}{V_g}(k_L \cdot e_L + k_C \cdot e_C) = \frac{V_o}{V_g} + \frac{1}{V_g} k^T \cdot e = \frac{V_o}{V_g} + u_e$$

Using this description in the error state-space equation results in

$$\dot{e} = \begin{bmatrix} 0 & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{RC} \end{bmatrix} e + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} u_e = A_e e + b_e u_e.$$

The last equation has the familiar form of a state-space equation with u_e as the manipulated variable. The state-space equation with $x = [i_L \quad v_o]^T$ does not fulfill this condition, because in that case the input variable u is the generator voltage V_g and the generator voltage is a disturbance variable. The state-space equation in e, in fact the matrix A_e and the vector b_e, enables the test of controllability. A block diagram of the system is shown in fig. 3.

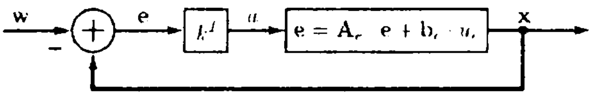

Fig. 3: Block diagram of a state-space controller

The structure in fig. 3 is nearly the same as in conventional control structures. An important difference to the block diagram in fig. 2 the setpoint w. In fig. 3 the steady state vector of the state variables is required. Because of the definition of the vector e the set value/actual value comparison is reversed. The controller design works with the characteristic polynomial of the closed loop

$$\dot{e} = A_e e + b_e u_e = A_e e + b_e k^T e = (A_e + b_e k^T)e.$$

The solution of this first order differential equation will go to zero, if the eigen-values have negative real parts. The calculation of the feedback parameters k_L and k_C is carried out via a comparison of the characteristic polynomial of the system and the pole placement.

$$\det(\underline{s}I - A - bk^T) = \underline{s}^2 + \underline{s}\left(\frac{1}{RC} - \frac{k_L}{L}\right) + \frac{1}{LC}\left(1 - \frac{k_L}{R} - k_C\right).$$

Example 1:
$L = 24\mu H$, $C = 40\mu F$, $R = 1.2\Omega$. The pole of the plant are at $\underline{s}_{\infty} = -10416\frac{1}{s} \pm j30548\frac{1}{s}$. If the poles of the controlled system should be at

$$\underline{s}^{*}_{\infty} = -30000\tfrac{1}{s} \pm j10000\tfrac{1}{s}$$

the feedback coefficients are $k_L = -0{,}94\,\Omega$ and $k_C = 0.8233$. A simulation of the averaged model is shown in fig. 4.

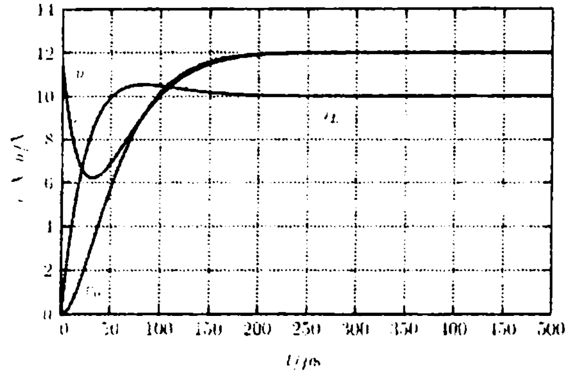

Fig 4: Step response of the set value: output voltage v_0, inductor current i_L, manipulated voltage u

Although a good controller design is available in small signal area, a DC error results in large signal area because of the proportional feedback [6]. Particularly the wave form of the inductor current has a large AC part. A second disadvantage is that the steady state load current has to be known. To avoid both disadvantages two measures are suggested. The output current can be taken as set point of the current and the inductor current can be filtered with a low pass filter. The accompanying signal flowchart is shown in fig. 5.

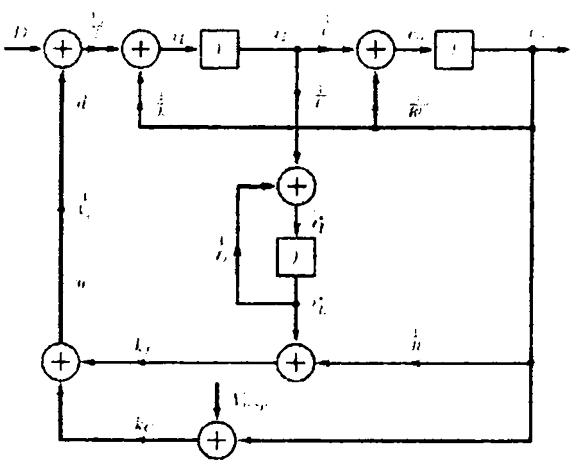

Fig. 5: Signal flowchart of the modified state-space controller

The signal flowchart in fig. 5 has the system matrix A_m ($x = \begin{bmatrix} i_L & i_L^* & v_o \end{bmatrix}^T$).

$$A_m = \begin{pmatrix} 0 & \frac{k_L}{L} & \frac{k_C - 1 - \frac{k_L}{R}}{L} \\ \frac{1}{T_1} & -\frac{1}{T_1} & 0 \\ \frac{1}{C} & 0 & -\frac{1}{RC} \end{pmatrix}.$$

The characteristic polynomial of this matrix is

$$N(\underline{s}) =$$
$$\underline{s}^3 + \underline{s}^2\left(\frac{1}{T_1} + \frac{1}{RC}\right) + \underline{s}\left(\frac{1}{RCT_1} + \frac{1}{LC} + \frac{k_L}{RLC} - \frac{k_C}{LC} - \frac{k_L}{LT_1}\right) + \frac{1}{LCT_1} - \frac{k_L}{LCT_1}.$$

The denominator of the control transfer function of the closed loop can be written in linear factors.

$$N(\underline{s}) = \left(\underline{s} - \underline{s}_{\infty,1}\right)\cdot\left(\underline{s} - \underline{s}_{\infty,2}\right)\cdot\left(\underline{s} - \underline{s}_{\infty,3}\right)$$

Comparing the two forms of the denominator gives

$$-\left(\underline{s}_{\infty,1} - \underline{s}_{\infty,2} - \underline{s}_{\infty,3}\right) = \frac{1}{T_1} + \frac{1}{RC}.$$

This equation shows that for the three poles of the denominator of the closed loop pole placement cannot be used. The dynamics of the closed loop depends on the choice of the time constant T_1 of the low pass filter and the load. In summary the easy form of a state-space controller showed in this chapter does not fulfill the requirements of a satisfactory controller design. So the easy state-space controller has to be modified in an other way to work robust and independent of load R and input voltage V_g.

IV. STATE-SPACE CONTROLLER WITH INNER ONE-CYCLE CONTROLLER AND OUTER I-CONTROLLER

In fig. 1 the low frequency equivalent circuit of the buck converter is shown. The voltage source $d \cdot V_g$ can be realized by an one-cycle controller (OCC), which integrates the input voltage of the converter [7]. An outer I-controller is added to improve robustness [4]. The structure is shown in fig. 6.

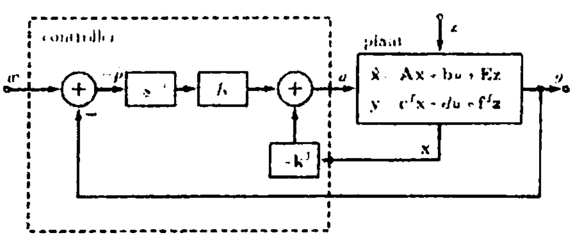

Fig. 6. State-space controller with outer I-controller and inner one-cycle controller

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The controller design equation is

$$\begin{bmatrix}\dot{x}\\ \dot{p}\end{bmatrix}=\begin{bmatrix}A-bk^T & -Kb\\ c^T-dk^T & -dK\end{bmatrix}\cdot\begin{bmatrix}x\\ p\end{bmatrix}+\begin{bmatrix}E & 0\\ f^T & -1\end{bmatrix}\cdot\begin{bmatrix}z\\ w\end{bmatrix}.$$

So the controller coefficients have the fulfill the characteristic equation of the closed loop.

$$\widetilde{P}(\underline{s})=\begin{bmatrix}\underline{s}I-A+bk^T & Kb\\ -c^T+dk^T & \underline{s}-dK\end{bmatrix}$$

In fig. 6 the plant is the buck converter including the one-cycle controller (OCC). The one-cycle controller causes an ideal feed forward of the input voltage and realizes a quasi linear voltage at the buck converter low pass filter. So the state-space controller design can be done as if there is a linear continuous amplifier. Because of the one-cycle controller and load description as a resistor the disturbance vector z is zero. The LC filter of the buck converter is a second order low pass filter. So the feed through factor d is zero. Considering these characteristics the dominator of the closed loop is calculated with

$$\widetilde{P}(\underline{s})=\begin{bmatrix}\underline{s}I-A+bk^T & Kb\\ -c^T & \underline{s}\end{bmatrix}.$$

Putting the matrices and vectors into the equation one gets

$$\widetilde{P}(\underline{s})=\begin{bmatrix}\underline{s}+\frac{k_L}{L} & \frac{1+k_C}{L} & \frac{K}{L}\\ -\frac{1}{C} & \underline{s}+\frac{1}{RC} & 0\\ 0 & -1 & \underline{s}\end{bmatrix}.$$

Example 2:
$L=24\,\mu\mathrm{H}$, $C=40\,\mu\mathrm{F}$, $R=1.2\,\Omega$, $f_{Clock}=100\,\mathrm{kHz}$.
The poles of the plant are at

$$\underline{s}_x=-10416\tfrac{1}{s}\pm j30548\tfrac{1}{s}.$$

Normalization with the constants $\omega_N=10^5\frac{1}{s}$ and $R_N=1\,\Omega$ gives the poles of the plant $\underline{s}_{x,N}=-0.10416\pm j0.30548$. If the normalized poles of the controlled system should be at $\underline{s}_x^*=-1.25$ the feedback coefficients are

$K=18.75\frac{1}{s}$, $k_l=8.5\,\Omega$ and $k_C=36.917$.

A simulation of the witch model is shown in fig. 7.

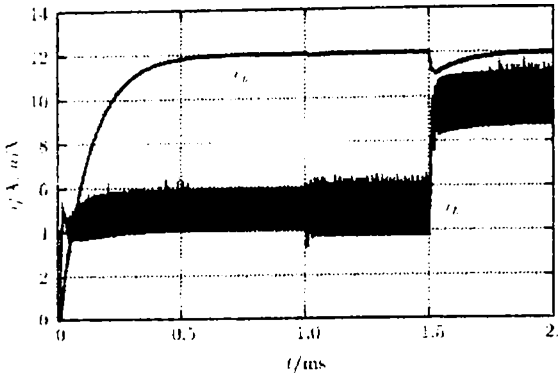

Fig. 7: Step response of the set value at $t=0$, step of the input voltage at $t=1\,\mathrm{ms}$ and load step at output voltage v_0, inductor current i_L

If the poles of the closed loop are too fast, there will arise a chaotic behavior. For example there is a Period-2 orbit for the normalized poles of the closed loop at $\underline{s}_x^*=-1.5$. This is typically for proportional feedback [9]. At $t=0.0015\,\mathrm{s}$ the input voltage has a step from $V_g=20\,\mathrm{V}$ to $V_g=25\,\mathrm{V}$. But there is nearly no effect to the output voltage because of the one-cycle controller. The very small effect, which can be seen in fig. 7. is an influence of the larger current ripple of the inductor and the proportional feedback. The ripple of the inductor current can be found in fig. 7. The dynamic of the output voltage is according to the pole placement. A load step is corrected in approximately $500\,\mu\mathrm{s}$. The dynamic is nearly independent from the load resistor, if not the inductor current is feed back but the difference of the inductor current and the load current. In that case we have a feedback of the capacitor current. Under steady state conditions the mean value of capacitor current is zero. So the current measurement can be realized by a transformer very efficiently. The large voltage drop at the load step is a consequence of the every small output capacitor.
The controller works very robust. Although an outer I-controller is added pole placement can be used. Because of the outer I-controller the steady state error is zero independent of the load R. The inner one-cycle controller causes an ideal feed forward of the input voltage V_g. Pole placement enables a very good damped and fast transients.

V. STATE-SPACE CONTROLLER FOR A BUCK CONVERTER WITH INPUT FILTER

The power stage circuit is shown in fig. 8

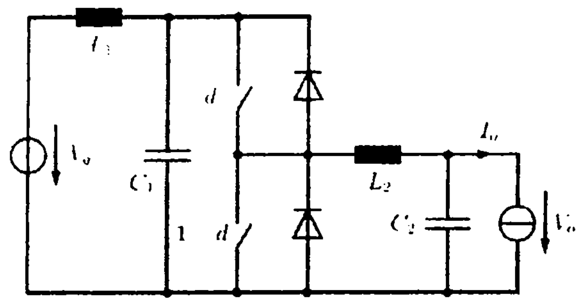

Fig. 8 Buck converter with undamped input filter

The control-to-output transfer function is

$$\frac{\tilde{v}_o}{\tilde{d}} = \frac{V_g \cdot \left(1 + \underline{s}^2 LC\right)}{1 + \underline{s}^2\left(L_1C_1 + L_1D^2C_2 + L_2C_2\right) + \underline{s}^4 L_1C_1L_2C_2}.$$

According to the characteristics of the equivalent circuit of the converter with a DC transformer the elements on the left side of the converter (L_1 and C_1) are considered by D^2 in this formula and there are two conjugated complex right half plane zeros because of the usually low damping factor. The same result can be obtained using Middlebrook's extra element theorem [8].

Example 3:
$V_g = 24\,\text{V}$, $L_1 = 50\,\mu\text{H}$, $C_1 = 100\,\mu\text{F}$, $V_o = 12\,\text{V}$, $f_{Clock} = 100\,\text{kHz}$. The buck converter output filter is $L_2 = 24\,\mu\text{H}$ and $C_2 = 40\,\mu\text{F}$. The load current is $I_o = 5\,\text{A}$ (fig. 8). Putting these values into the equation above one gets

$$\frac{\tilde{v}_o}{\tilde{d}} = \frac{V_g \cdot \left(1 + \underline{s}^2 \cdot 5 \cdot 10^{-9}\,\text{s}^2\right)}{1 + \underline{s}^2 \cdot 6.46 \cdot 10^{-9}\,\text{s}^2 + \underline{s}^4 \cdot 4.8 \cdot 10^{-18}\,\text{s}^4}.$$

The circuit is undamped. So the pole points are on the imaginary axis. The poles are $\underline{s}_{\infty,3/4} = \pm j34167\,\frac{1}{\text{s}}$ and $\underline{s}_{\infty,3/4} = \pm j13359\,\frac{1}{\text{s}}$. The zeros are $\underline{s}_{0,1,2} = \pm j14142\,\frac{1}{\text{s}}$. The bode plot of the control-to-output transfer function is shown in fig. 9.

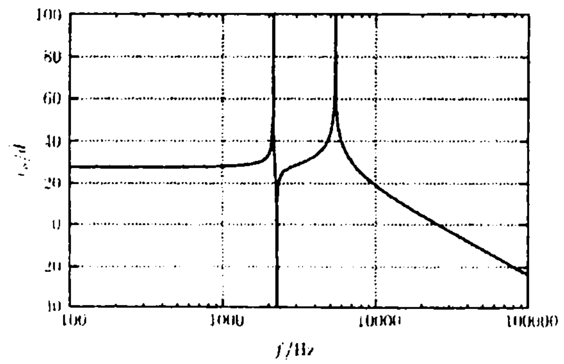

Fig. 9: Bode plot of the control-to-output transfer function

The buck converter in fig. 8 can be written in state-space with the errors of the inductor currents and capacitor voltages (see chapter 2)[5]. We will get

$$\begin{bmatrix} \dot{e}_{L1} \\ \dot{e}_{C1} \\ \dot{e}_{L2} \\ \dot{e}_{C2} \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{L_1} & 0 & 0 \\ \frac{1}{C} & 0 & -\frac{D_{sp}}{C_1} & 0 \\ 0 & \frac{D_{sp}}{L_2} & 0 & -\frac{1}{L_2} \\ 0 & 0 & \frac{1}{C_2} & 0 \end{bmatrix} \cdot \begin{bmatrix} e_{L1} \\ e_{C1} \\ e_{L2} \\ e_{C2} \end{bmatrix} + \begin{bmatrix} 0 \\ -\frac{I_o}{C_1V_g} \\ \frac{1}{L_2} \\ 0 \end{bmatrix} \cdot u \cdot$$

In this equation are $D_{sp} = V_{sp} / V_g$, $u = \boldsymbol{k}^T\boldsymbol{e}$, $e_{L1} = i_{L1} - V_{sp}^2 / (V_g R)$, $e_{L2} = i_{L2} - V_{sp} / R$, $e_{C1} = v_{C1} - V_g$, $e_{L2} = i_{L2} - V_{sp} / R$, $e_{C2} = v_{C2} - V_{sp}$, $\boldsymbol{k}^T = \begin{bmatrix} k_{L1} & k_{C1} & k_{L2} & k_{C2} \end{bmatrix}$ and V_{sp} the set point of the voltage. It can be shown that the system is controllable [5]. The denominator of the closed loop with state feedback can be calculated with the determinate $P(s) = \det\left|\underline{s}\,\boldsymbol{I} - \boldsymbol{A}_e - \boldsymbol{b}\,\boldsymbol{k}^T\right|$.

$$\begin{aligned} P(\underline{s}) = \underline{s}^4 &+ \frac{\underline{s}^3}{C_1C_2L_1L_2V_g}\left(C_2L_1L_2I_ok_{C1} - C_1C_2L_1V_gk_{L2}\right) + \\ &\frac{\underline{s}^2}{C_1C_2L_1L_2V_g}\begin{pmatrix} C_1L_1V_g - C_1L_1V_gk_{C2} + C_2L_1D_{sp}^2V_g + C_2L_2V_g + \\ C_2L_1D_{sp}I_ok_{L2} + C_2L_1D_{sp}V_gk_{C1} - C_2L_2I_ok_{L1} \end{pmatrix} + \\ &\frac{\underline{s}}{C_1C_2L_1L_2V_g}\left(L_1I_ok_{C1} + L_1D_{sp}I_ok_{C2} - C_2V_gk_{L2} - C_2D_{sp}V_gk_{L1}\right) + \\ &\frac{1}{C_1C_2L_1L_2V_g}\left(V_g - V_gk_{C2} - I_ok_{C1}\right) \end{aligned}$$

This polynomial has to fulfill the pole placement. So the polynomial of the closed loop can be written as

$$P(\underline{s}_N) = \underline{s}_N^4 + c_3\,\underline{s}_N^3 + c_2\,\underline{s}_N^2 + c_1\,\underline{s}_N + c_0.$$

Comparing the two polynomials results in solving a linear system of equations for calculating the feedback coefficients of the controller.

Example 4:
The buck converter according fig. 8 is considered with the values used in example 3. Putting these values into the equations above and normalizing with the constants $\omega_N = 10^4\,\frac{1}{\text{s}}$ and $R_N = 1\,\Omega$ one gets

$$\begin{aligned} P(\underline{s}) = \underline{s}^4 &+ \frac{\underline{s}^3}{1.152}\left(0.24k_{C1} - 4.8k_{L2}\right) + \\ &\frac{\underline{s}^2}{1.152}\left(15.504 - 2.4k_{C1} + 0.48k_{L1} + 12k_{C2} + 0.5k_{L2}\right) + \\ &\frac{\underline{s}}{1.152}\left(2.5k_{C1} + 4.8k_{L1} - 1.25k_{C2} - 9.6k_{L2}\right) + \\ &\frac{1}{1.152}\left(24 - 5k_{L1} - 24k_{C2}\right) \end{aligned}$$

A good pole placement for a damped step response is $\tilde{\underline{s}}_{N,\infty,1/2} = -1$ and $\tilde{\underline{s}}_{N,\infty,3/4} = -4$. The denominator of the closed loo becomes

$$\tilde{P}(\underline{s}_N) = \underline{s}_N^4 + 10\,\underline{s}_N^3 + 33\,\underline{s}_N^2 + 40\,\underline{s}_N + 16.$$

The linear system of equations can be written in matrix form.

$$\begin{bmatrix} 0.20833 & 0.0 & 0.0 & -4.16666 \\ 2.0833 & -0.41666 & -10.4166 & 0.43403 \\ -2.17101 & -4.16666 & 1.08507 & -8.33333 \\ 0.0 & 4.34028 & -20.8333 & 0.0 \end{bmatrix} \begin{bmatrix} k_{L1} \\ k_{C1} \\ k_{L2} \\ k_{C2} \end{bmatrix} = \begin{bmatrix} 10.0 \\ 19.542 \\ 40.0 \\ -4.8333 \end{bmatrix}$$

Solving this system of equations yields

$$\begin{bmatrix} k_{L1} \\ k_{C1} \\ k_{L2} \\ k_{C2} \end{bmatrix} = \begin{bmatrix} 10.521 \\ -0.33 \\ 0.16326 \\ -1.8739 \end{bmatrix}.$$

This vector is the feedback vector of the errors of the state variables of the system. Under steady state conditions the converter works well with a smooth input current and a smooth output voltage (fig. 10). Disadvantages are a difficult controller design and the dependency of the feedback coefficients of the input voltage and the output current.

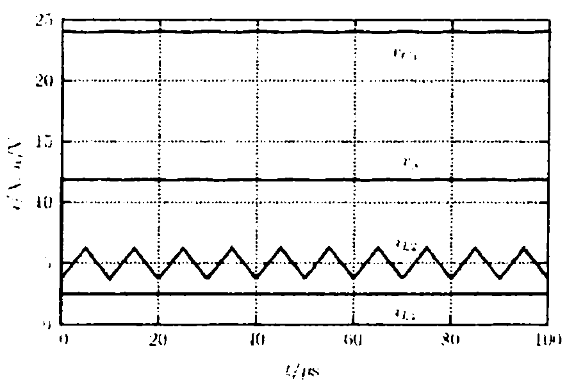

Fig 10: Steady state operation of the buck converter with input filter: capacitor voltage v_{C1}, output voltage v_o, inductor current v_{L2}, input current v_{L1}

VI. SUMMARY

State-space controllers can be applied to buck converters using linear models of the power stage. Because of the proportional feedback of the state variables an outer I-controller is recommended to avoid steady state error. For a good audiosusceptibility an inner one-cycle controller is added. This one-cycle controller works as a feed forward of the input voltage.

State-space controllers are predestinated to control high order systems. Such systems are buck converters with an input filter. Because of consideration of the input filter at the design step of the controller, there will be no interaction between input filter and converter, even though the input filter elements are much smaller than in conventional designs [1].

The advantages of the proposed controller are a smaller size, lower costs and higher efficiency of the converter, compared with a converter with a conventional input filter, which is damped by a resistance. Disadvantages are a difficult controller design and the dependency of the feedback coefficients of the input voltage and the output current.

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