

Tom 49 (63), Fascicola 2, 2004

**Denoising Over-Sampled Signals**André Quinquis<sup>1</sup>, Alexandru Isar<sup>2</sup>, IEEE Members and Dorina Isar<sup>2</sup>

**Abstract** - This paper presents a new denoising method for over-sampled constant within intervals signals corrupted by additive noise. The novelty of this paper is a special MAP filter, called composed bishrink. A complete statistical analysis of this filter is reported. Some simulations are presented. The results obtained are compared with the results of other denoising methods and with other state-of-the-art filtering techniques.

**Keywords:** wavelets, denoising, soft thresholding, bishrink.

## I. INTRODUCTION

In recent years, the techniques that use multiscale and local transform-based algorithms have become popular in noise filtering applications. In particular the use of non-linear filters in the DCT domain was studied, [1]. In this paper, we consider local transform based denoising. We propose such an algorithm, based on the discrete wavelet transform, (DWT). Section II deals with the local DWT-based denoising. In section III, a statistical analysis of the new denoising method is presented. The use of local filters in the DWT domain is described in the following section. In section V, numerical simulation results are presented and discussed. The last section is dedicated to some concluding remarks.

## II. LOCAL DWT-BASED DENOISING

The following model of the observed signal corrupted by additive noise is considered in this paper:

$$x[k] = s[k] + n[k] \quad (1)$$

where  $s$  and  $n$  represent the useful part and the noise. The problem is to estimate  $s$  starting from  $x$ . The noise is usually considered to be uncorrelated with  $s$ , stationary random process, with a null mean and a variance  $\sigma_n^2$ . To estimate the signal  $s$ , Donoho, [2], proposed the following method:

1. The Discrete Wavelet Transform (DWT) of the signal  $x$  is computed. The result is the signal  $y_i = y + n_i$ . The noise  $n_i$  converges asymptotically to a Gaussian white one, with the same variance, [3].

2. A non-linear filtering is applied in the wavelet domain:

where  $t$  is a threshold. This system is called soft

$$y_0[k] = \begin{cases} \text{sgn}\{y_i[k]\}(|y_i[k]| - t), & |y_i[k]| > t \\ 0, & \text{if not} \end{cases} \quad (2)$$

thresholding filter. Because the noise  $n_i$  is Gaussian, if  $t > 3\sigma_n$ , the probability  $P(n_i > t)$  is very little (the rule of 3 sigmas). So the noise is quasi entirely suppressed. This is the reason why the signal  $y_0$  is a denoised version of the signal  $y_i$ . This is a non-linear adaptive filter whose statistic analysis was presented in [4]. The adaptability is due to the selection of the threshold value in function of the noise power.

3. Taking the inverse DWT (IDWT) of the signal  $y_0$ , the denoised version of the signal  $s$ ,  $x_0$ , is obtained.

The principal disadvantage of the already described denoising method is due to the fact that it is based only on the estimation of the noise variance (the useful part of the input signal is ignored) and on hypotheses confirmed only asymptotically. This is the reason why in the following another denoising strategy, based on the use of a Maximum a Posteriori, MAP, filter, will be described.

## III. A STATISTICAL ANALYSIS OF THE DWT

The probability density function, (pdf), of the wavelet coefficients at the  $m$ 'th scale, (after  $m$  iterations),  ${}_x D_m^k$  ( $k$  being equal with 1 for detail coefficients and with 2 for approximation coefficients) is given by the following relation:

$$f_{{}_x D_m^k}(a) = \begin{matrix} N(k) M_0 \\ * & * & \dots \\ \eta_1 = 1 & \eta_2 = 1 \\ M_0 \\ * & f_d(k, \eta_1, \eta_2, \dots, \eta_m, a) \\ q_m = 1 \end{matrix} \quad (3)$$

where:

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