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Optical coherent and incoherent systems frequency analyze in Cartesian coordinate

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Abstract-Our goal in this paper is to make a study about optical coherent and incoherent systems frequency response. We begin from the definition of an optical system then we define transfer function for coherent and incoherent systems. We find the response of this system to a step indices stimulus, then we generalize for an image and finally we make a comparison between two different systems.

INTRODUCTION

Generally speaking optical systems can be seen as a black box with an input and an output. at the input we have object plane and at the output image plane which is obtained from convolution between object image and transfer function of optical systems (the black box can contain one ore more optical elements). Here we will have a diffraction element.

$$U_i(u,v) = \iint_{\infty} h(u-\xi; v-\eta) U_o(\xi,\eta) d\xi d\eta \quad (1)$$

$h(u,v;\xi,\eta)$ optical system impulse response

but system response to optical impulse is Fourier transform (Fraunhoffer diffraction) of diffraction element aperture.

$$h(u,v) = \frac{A}{\lambda z_i} \iint P(x,y) \exp\{-j\frac{2\pi}{\lambda z_i}(ux+vy)\} dxdy \quad (2)$$

Next we will try to calculate *h(u,v)* for coherent and incoherent case.

What do we understand by coherent and incoherent illumination?

Coherent illumination is made by lasers.

Incoherent illumination is made by diffuse source like sun or gaze lamp.

For coherent illumination the system is described by amplitude convolution equation.

$$U_i(u,v) = \iint_{\infty} h(u-\xi; v-\eta) U_o(\xi,\eta) d\xi d\eta \quad (3)$$

For incoherent illumination the system is described by intensity convolution equation.

$$I_i = \iint |h(u-\xi, v-\eta)|^2 I_o(\xi,\eta) d\xi d\eta \quad (4)$$

AMPLITUDE TRANSFER FUNCTIONS TYPICAL FOR COHERENT CASE.

We define input and output frequency spectrum

$$G_o(f_x,f_y) = \iint U_o(x,y) \exp\{-j2\pi(f_x u + f_y v)\} dvdu \quad (5)$$

$$G_i(f_x,f_y) = \iint U_i(x,y) \exp\{-j2\pi(f_x u + f_y v)\} dvdu \quad (6)$$

We define transfer function

$$H_i(f_x,f_y) = \iint h(u,v) \exp\{-j2\pi(f_x u + f_y v)\} dvdu \quad (7)$$

We apply convolution to (3) and we obtain:

$$G_i(f_x,f_y) = H(f_x,f_y) G_o(f_x,f_y) \quad (8)$$

This is the relation between image and object plane in frequency.

But transfer function is Fourier Transform of impulse response system. Then we will have:

$$H(f_x,f_y) = F\{\frac{A}{\lambda z_i} \iint P(x,y) \exp\{-j\frac{2\pi}{\lambda z_i}(ux+vy)\} dxdy\}$$
$$= (A\lambda z_i) P(\lambda z_i f_x, \lambda z_i f_y) \quad (9)$$

If we put $A\lambda z_i = 1$ then

$$H(f_x,f_y) = P(\lambda z_i f_x, \lambda z_i f_y) \quad (10)$$

As a conclusion for coherent illumination Amplitude Transfer Function is the aperture trough which the light passes and the diffraction is made. For a square aperture we will have:

P(x,y)= $rect(\frac{x}{2w}) rect(\frac{y}{2w})$

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The transfer function will be:

$$H(f_x, f_y) = rect\left(\frac{\lambda z_i f_x}{2w}\right) rect\left(\frac{\lambda z_i f_y}{2w}\right)$$

Next we will study optical coherent systems response to a step indices stimulus for a square aperture. We will study 2D and 3D case Fig. 1

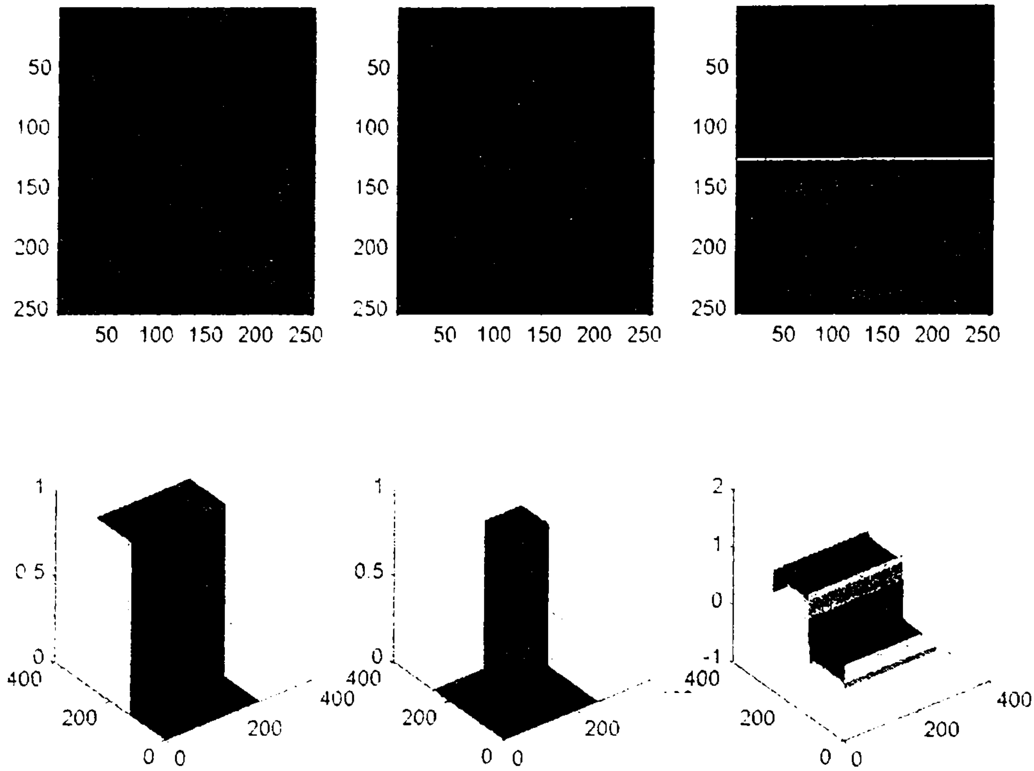

Fig. 1. first line present 2D case: second line present 3D case; first column present step indices stimulus; second line present square aperture: third line present response

OPTICAL TRANSFER FUNCTION TYPICAL FOR INCOHERENT CASE

We define normalized frequency spectrum I_i and I_o:

$$G_o(f_x, f_y) = \frac{\iint I_o(u,v)\exp[-j2\pi(f_x u + f_y v)]dudv}{\iint I_o(u,v)dudv} \quad (11)$$

$$G_i(f_x, f_y) = \frac{\iint I_i(u,v)\exp[-j2\pi(f_x u + f_y v)]dudv}{\iint I_i(u,v)dudv} \quad (12)$$

We define transfer function

$$H(f_x, f_y) = \frac{\iint |h(uv)|^2 \exp[-j2\pi(f_x u + f_y v)]dudv}{\iint |h(u,v)|^2 dudv} \quad (13)$$

We apply convolution to (4) and we obtain:

$$G_i(f_x, f_y) = H(f_x, f_y) G_o(f_x, f_y) \quad (14)$$

H(f_x, f_y) optical transfer function

Optical transfer function and optical amplitude function on their definition imply function h (optical system impulse response) so there is a relation between this two function. Optical transfer function is the normalized autocorrelation of amplitude transfer function.

$$H(f_x, f_y) = \frac{\iint P(x + \frac{\lambda z_i f_x}{2}, y + \frac{\lambda z_i f_y}{2}) P(x - \frac{\lambda z_i f_x}{2}, y - \frac{\lambda z_i f_y}{2}) dxdy}{\iint P(x, y) dxdy} \quad (15)$$

This represent area of superposition of two apertures of the same shape one at $\frac{\lambda z_i f_x}{2}, \frac{\lambda z_i f_y}{2}$ the other at $-\frac{\lambda z_i f_x}{2}, -\frac{\lambda z_i f_y}{2}$ divided at total area of the two apertures as in Fig. 2

Mathematical relation of common area is:

$$A(f_x, f_y) = \begin{cases} (2w - \lambda z_i |f_x|)(2w - \lambda z_i |f_y|) \\ 0 \end{cases}$$

$$|f_x| \le \frac{2w}{\lambda z}, |f_y| \le \frac{2w}{\lambda z}$$

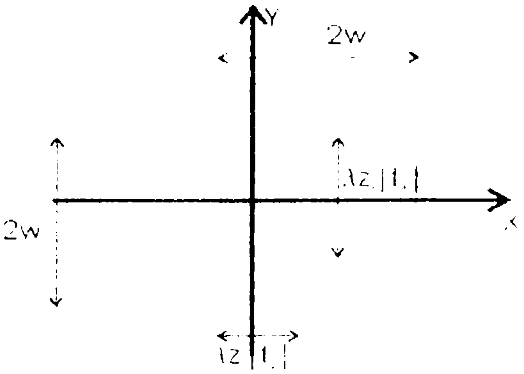

Fig. 2 $H(f_x, f_y)$ for incoherent case.

When this area is normalized with total area $4w^2$ have

$$H(f_x, f_y) = tri(\frac{f_x}{2f_0}) tri(\frac{f_y}{2f_0})$$

$f_0 = \frac{w}{\lambda z}$ cutoff frequency for coherent case

Next we will study optical incoherent systems response to a step indices stimulus for a triangular aperture. We will study 2D and 3D case Fig. 3

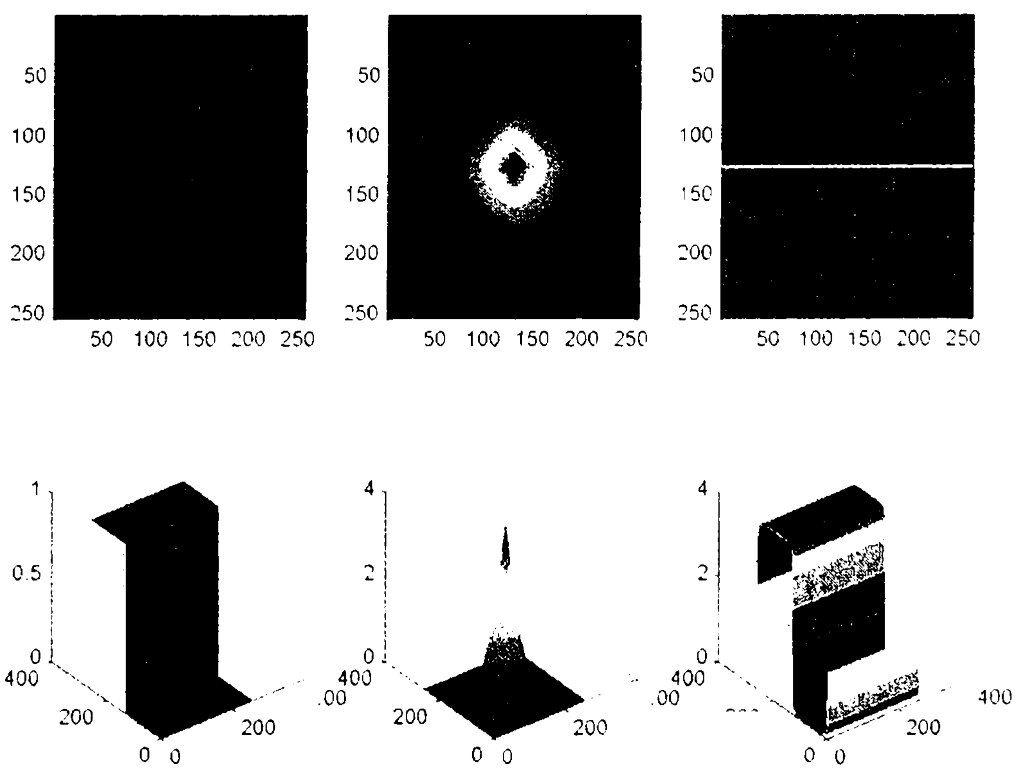

Fig. 3. first line present 2D case; second line present 3D case; first column present step indices stimulus; second line present triangle aperture; third line present response .

CONCLUSION

Comparing response in Fig 1 and Fig 3 we see a great difference between optical coherent and incoherent system response for a step indices. So for optical coherent systems we have a response with oscillation at the end (Gibb phenomenon) and a phase difference from the axe of symmetry. Optical incoherent systems do not have oscillation at the end and phase difference. Finally to have a clear view we will put an image instead of step indices stimulus and well see how acts in the two cases. Fig.4

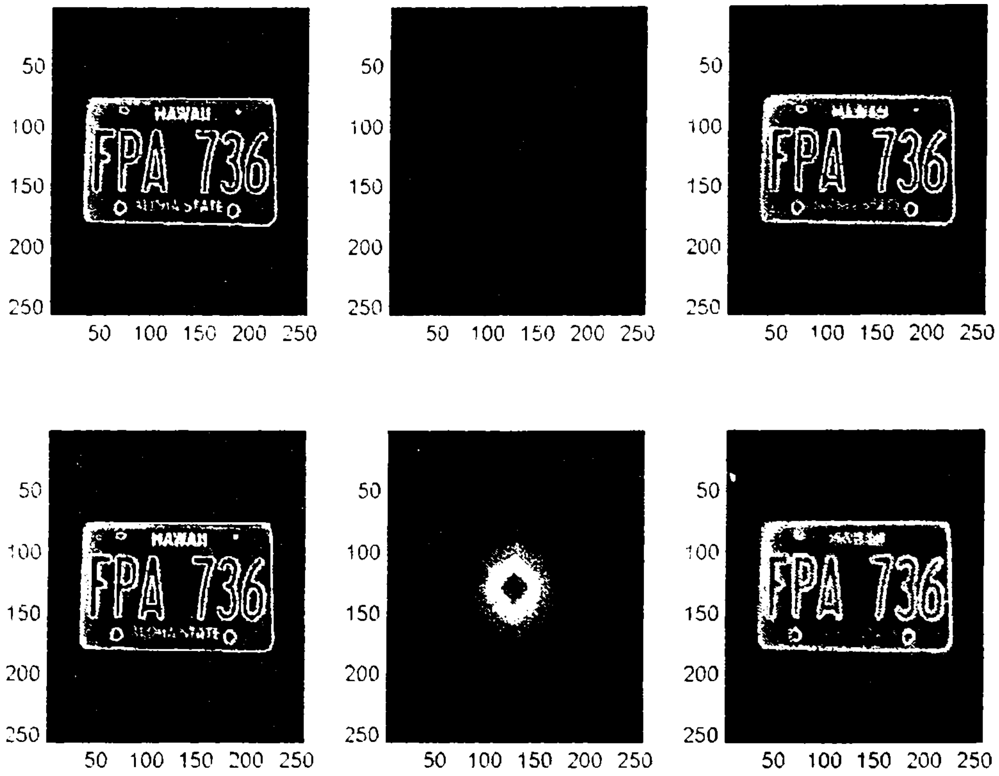

Fig. 4. first line present coherent case; second line present incoherent case; first column present input image; second line present square aperture and triangle aperture; third line present output image.

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