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LYAPUNOV FUNCTIONALS FOR SKEW-EVOLUTION SEMIFLOWS IN BANACH SPACES

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Abstract

The paper considers a notion of nonuniform splitting with growth rates for skew-evolution semiflows in Banach spaces. Characterizations for this concept are given through Lyapunov functionals with invariant and strongly invariant families of projections. [1]

Keywords and phrases: *Lyapunov functionals, skew-evolution semiflows, splitting*

1 Introduction. Preliminaries

The asymptotic property of (exponential) splitting was introduced by B. Aulbach and J. Kalkbrenner in [1] as a generalization of (exponential) dichotomy for difference equations. Regarding the qualitative results obtained for the dichotomy notion, we mention the contributions from [2], [4], [6] and the references therein.

Recent studies for more general concepts of splitting are made in [3] for noninvertible differential equations with impulse effect, respectively in [5] for skew-evolution semiflows.

The integral conditions represent an important tool to give criteria for asymptotic behaviours (see for instance [7], [8]). In this article, a result for nonuniform splitting with Lyapunov functionals is proved from the point of view of invariant families of projections, using an auxiliary integral characterization. Also, similar results are shown in the case of strongly invariant families of projections.

[1]MSC(2008): 34D05, 93D30

Let X be a real or complex Banach space and Θ a metric space. $\mathcal{B}(X)$ represents the Banach algebra of all bounded linear operators on X and the norms on X, respectively on $\mathcal{B}(X)$, will be denoted by $||\cdot||$. We consider the sets

$$\Delta = \{(t,s) \in \mathbb{R}_+^2 : t \geq s\}, \quad T = \{(t,s,t_0) \in \mathbb{R}_+^3 : t \geq s \geq t_0\}$$

and $\Gamma = \Theta \times X$.

Definition 1.1. A continuous mapping $\varphi : \Delta \times \Theta \to \Theta$ is called *evolution semiflow* on Θ if the following relations hold:

(es_1) $\varphi(s,s,\theta) = \theta,$ for all $(s,\theta) \in \mathbb{R}_+ \times \Theta;$

(es_2) $\varphi(t,s,\varphi(s,t_0,\theta)) = \varphi(t,t_0,\theta),$ for all $(t,s,t_0,\theta) \in T \times \Theta.$

Definition 1.2. We say that $\Phi : \Delta \times \Theta \to \mathcal{B}(X)$ is *evolution cocycle* over the evolution semiflow φ if:

(ec_1) $\Phi(s,s,\theta) = I$ (the identity operator on X), for all $(s,\theta) \in \mathbb{R}_+ \times \Theta;$

(ec_2) $\Phi(t,s,\varphi(s,t_0,\theta))\Phi(s,t_0,\theta) = \Phi(t,t_0,\theta),$ for all $(t,s,t_0,\theta) \in T \times \Theta;$

(ec_3) $(t,s,\theta) \mapsto \Phi(t,s,\theta)x$ is continuous for every $x \in X.$

Definition 1.3. If φ is evolution semiflow on Θ and Φ is evolution cocycle over the evolution semiflow φ, then the pair $C = (\varphi, \Phi)$ is called *skew-evolution semiflow* on Γ.

Definition 1.4. A continuous mapping $P : \mathbb{R}_+ \times \Theta \to \mathcal{B}(X)$, which satisfies

$$P^2(t,\theta) = P(t,\theta), \quad \text{for all} \quad (t,\theta) \in \mathbb{R}_+ \times \Theta,$$

is called *family of projections* on X.

If $P : \mathbb{R}_+ \times \Theta \to \mathcal{B}(X)$ is a family of projections, then $Q : \mathbb{R}_+ \times \Theta \to \mathcal{B}(X)$, $Q(t,\theta) = I - P(t,\theta)$ is the *complementary family of projections of* P.

Definition 1.5. A family of projections $P : \mathbb{R}_+ \times \Theta \to \mathcal{B}(X)$ is called *invariant* for the skew-evolution semiflow $C = (\varphi, \Phi)$ if:

$$P(t,\varphi(t,s,\theta))\Phi(t,s,\theta) = \Phi(t,s,\theta)P(s,\theta), \quad \text{for all } (t,s,\theta) \in \Delta \times \Theta.$$

If in addition, for all $(t,s,\theta) \in \Delta \times \Theta$ the mapping $\Phi(t,s,\theta)$ is an isomorphism from *Range* $Q(s,\theta)$ to *Range* $Q(t,\varphi(t,s,\theta))$, then we say that P is *strongly invariant for* $C = (\varphi, \Phi)$.

Let $C = (\varphi, \Phi)$ be a skew-evolution semiflow, $P : \mathbb{R}_+ \times \Theta \to \mathcal{B}(X)$ an invariant family of projections for C and $h, k : \mathbb{R}_+ \to [1, +\infty)$ growth rates (i.e. nondecreasing functions with $\lim\limits_{t\to+\infty} h(t) = \lim\limits_{t\to+\infty} k(t) = +\infty$).

Definition 1.6. The pair (C, P) admits (h, k)-*splitting* if there exist $\alpha, \beta \in \mathbb{R}$, with $\alpha < \beta$ and a nondecreasing function $N : \mathbb{R}_+ \to [1, +\infty)$ such that:

(hs_1) $h(s)^\alpha ||\Phi(t, t_0, \theta)P(t_0, \theta)x|| \leq N(s)h(t)^\alpha ||\Phi(s, t_0, \theta)P(t_0, \theta)x||$;

(ks_1) $k(t)^\beta ||\Phi(s, t_0, \theta)Q(t_0, \theta)x|| \leq N(t)k(s)^\beta ||\Phi(t, t_0, \theta)Q(t_0, \theta)x||$,

for all $(t, s, t_0, \theta, x) \in T \times \Gamma$.

In particular, if $\alpha < 0 < \beta$, then we have the concept of (h, k)-*dichotomy*.

Definition 1.7. We say that (C, P) has (h, k)-*growth* if there exist $\omega > 0$ and a nondecreasing function $M : \mathbb{R}_+ \to [1, +\infty)$ with:

(hg_1) $h(s)^\omega ||\Phi(t, t_0, \theta)P(t_0, \theta)x|| \leq M(t_0)h(t)^\omega ||\Phi(s, t_0, \theta)P(t_0, \theta)x||$;

(kg_1) $k(s)^\omega ||\Phi(s, t_0, \theta)Q(t_0, \theta)x|| \leq M(t)k(t)^\omega ||\Phi(t, t_0, \theta)Q(t_0, \theta)x||$,

for all $(t, s, t_0, \theta, x) \in T \times \Gamma$.

Further, we recall some results obtained in [5].

Proposition 1.1. *If* $P : \mathbb{R}_+ \times \Theta \to \mathcal{B}(X)$ *is a strongly invariant family of projections for* $C = (\varphi, \Phi)$*, then there exists an isomorphism* $\Psi : \Delta \times \Theta \to \mathcal{B}(X)$ *from Range* $Q(t, \varphi(t, s, \theta))$ *to Range* $Q(s, \theta)$*, such that:*

(Ψ^1) $\Phi(t, s, \theta)\Psi(t, s, \theta)Q(t, \varphi(t, s, \theta)) = Q(t, \varphi(t, s, \theta))$;

(Ψ^2) $\Psi(t, s, \theta)\Phi(t, s, \theta)Q(s, \theta) = Q(s, \theta)$;

(Ψ^3) $\Psi(t, s, \theta)Q(t, \varphi(t, s, \theta)) = Q(s, \theta)\Psi(t, s, \theta)Q(t, \varphi(t, s, \theta))$;

(Ψ^4) $\Psi(t, t_0, \theta)Q(t, \varphi(t, t_0, \theta)) = \Psi(s, t_0, \theta)\Psi(t, s, \varphi(s, t_0, \theta))Q(t, \varphi(t, t_0, \theta))$,

for all $(t, s, t_0, \theta) \in T \times \Theta$.

Proof. See [5], Proposition 2.9. □

We denote by $\mathcal{H}_1$ the set of all growth rates $h : \mathbb{R}_+ \to [1, +\infty)$ with

$$\int_0^{+\infty} h(s)^c ds < +\infty, \quad \text{for all } c < 0.$$

Also, $\mathcal{K}_1$ represents the set of all growth rates $k : \mathbb{R}_+ \to [1, +\infty)$, with the property that there exists a constant $K_1 \geq 1$ such that

$$\int_0^t k(s)^c ds \leq K_1 k(t)^c, \quad \text{for all } c > 0,\ t \geq 0.$$

By $\mathcal{H}$ we denote the set of all growth rates $h : \mathbb{R}_+ \to [1, +\infty)$ with the property that there exists $H > 1$ such that

$$1 \leq \frac{h(t+1)}{h(t)} < H, \quad \text{for all} \quad t \geq 0.$$

Theorem 1.1. *Let (C, P) be a pair with (h, k)-growth, where $h \in \mathcal{H}_1 \cap \mathcal{H}$ and $k \in \mathcal{K}_1 \cap \mathcal{H}$. Then (C, P) admits (h, k)-splitting if and only if there exist $d_1, d_2 \in \mathbb{R}$, $d_1 < d_2$ and a nondecreasing mapping $D : \mathbb{R}_+ \to [1, +\infty)$ such that the following assertions hold:*

$$(Dhs_1) \qquad \int_s^{+\infty} \frac{||\Phi(\tau, t_0, \theta)P(t_0, \theta)x||}{h(\tau)^{d_1}} d\tau \leq \frac{D(s)}{h(s)^{d_1}} ||\Phi(s, t_0, \theta)P(t_0, \theta)x||,$$

for all $(s, t_0, \theta, x) \in \Delta \times \Gamma$;

$$(Dks_1) \qquad \int_{t_0}^{t} \frac{||\Phi(\tau, t_0, \theta)Q(t_0, \theta)x||}{k(\tau)^{d_2}} d\tau \leq \frac{D(t)}{k(t)^{d_2}} ||\Phi(t, t_0, \theta)Q(t_0, \theta)x||,$$

for all $(t, t_0, \theta, x) \in \Delta \times \Gamma$.

Proof. See [5], Theorem 3.2. □

2 The main results

Let $C = (\varphi, \Phi)$ be a skew-evolution semiflow, $P : \mathbb{R}_+ \times \Theta \to \mathcal{B}(X)$ an invariant family of projections for C and $h, k : \mathbb{R}_+ \to [1, +\infty)$ two growth rates.

Definition 2.1. We say that $L : T \times \Gamma \to \mathbb{R}_+$ is (h, k)-*Lyapunov functional* for the pair (C, P) if there exist two real constants $l_1 < l_2$ such that:

$$(hL_1) \quad \int_s^t \frac{||\Phi(\tau, t_0, \theta)P(t_0, \theta)x||}{h(\tau)^{l_1}} d\tau \leq \frac{L(s, s, t_0, \theta, P(t_0, \theta)x) - L(t, s, t_0, \theta, P(t_0, \theta)x)}{h(s)^{l_1}};$$

$$(kL_1) \quad \int_s^t \frac{||\Phi(\tau, t_0, \theta)Q(t_0, \theta)x||}{k(\tau)^{l_2}} d\tau \leq \frac{L(t, t, t_0, \theta, Q(t_0, \theta)x) - L(t, s, t_0, \theta, Q(t_0, \theta)x)}{k(t)^{l_2}},$$

for all $(t, s, t_0, \theta, x) \in T \times \Gamma$.

Theorem 2.1. *We consider (C, P) a pair with (h, k)-growth, where $h \in \mathcal{H}_1 \cap \mathcal{H}$ and $k \in \mathcal{K}_1 \cap \mathcal{H}$. Then (C, P) admits (h, k)-splitting if and only if there exist $L : T \times \Gamma \to \mathbb{R}_+$ a (h, k)-Lyapunov functional for (C, P) and a nondecreasing function $\lambda : \mathbb{R}_+ \to [1, +\infty)$ with:*

(L_1) $L(s, s, t_0, \theta, P(t_0, \theta)x) \leq \lambda(s)||\Phi(s, t_0, \theta)P(t_0, \theta)x||$;

(L_2) $L(t, t, t_0, \theta, Q(t_0, \theta)x) \leq \lambda(t)||\Phi(t, t_0, \theta)Q(t_0, \theta)x||$,

for all $(t, s, t_0, \theta, x) \in T \times \Gamma$.

Proof. Necessity. Let $L : T \times \Gamma \to \mathbb{R}_+$ be defined by

$$L(t, s, t_0, \theta, x) = \int_t^{+\infty} \left(\frac{h(s)}{h(\tau)}\right)^{d_1} ||\Phi(\tau, t_0, \theta)P(t_0, \theta)x||d\tau +$$

$$+ \int_{t_0}^{s} \left(\frac{k(t)}{k(\tau)}\right)^{d_2} ||\Phi(\tau, t_0, \theta)Q(t_0, \theta)x||d\tau,$$

where $d_1 < d_2$ are given by Theorem 1.1.
It is immediate to see that the (hL_1) and (kL_1) from Definition 2.1 are satisfied. From Theorem 1.1, we deduce that (L_1) and (L_2) are verified.

Sufficiency. Using Definition 2.1, (hL_1), we have

$$\int_s^t \frac{||\Phi(\tau, t_0, \theta)P(t_0, \theta)x||}{h(\tau)^{l_1}} d\tau \leq \frac{L(s, s, t_0, \theta, P(t_0, \theta)x)}{h(s)^{l_1}} \leq$$

$$\leq \frac{\lambda(s)}{h(s)^{l_1}} ||\Phi(s, t_0, \theta)P(t_0, \theta)x||,$$

which implies

$$\int_s^{+\infty} \frac{||\Phi(\tau, t_0, \theta)P(t_0, \theta)x||}{h(\tau)^{l_1}} d\tau \leq \frac{\lambda(s)}{h(s)^{l_1}} ||\Phi(s, t_0, \theta)P(t_0, \theta)x||, \tag{1}$$

for all $(s, t_0, \theta, x) \in \Delta \times \Gamma$.
Similarly, from (kL_1), for $t_0 = s$ it follows

$$\int_{t_0}^{t} \frac{||\Phi(\tau,t_0,\theta)Q(t_0,\theta)x||}{k(\tau)^{l_2}} d\tau \leq \frac{L(t,t,t_0,\theta,Q(t_0,\theta)x)}{k(t)^{l_2}}$$

and then

$$\int_{t_0}^{t} \frac{||\Phi(\tau,t_0,\theta)Q(t_0,\theta)x||}{k(\tau)^{l_2}} d\tau \leq \frac{\lambda(t)||\Phi(t,t_0,\theta)Q(t_0,\theta)x||}{k(t)^{l_2}}, \tag{2}$$

for all $(t,t_0,\theta,x) \in \Delta \times \Gamma$.
From (1), (2) and Theorem 1.1 we obtain that (C,P) has (h,k)-splitting. □

In what follows, $P : \mathbb{R}_+ \times \Theta \to \mathcal{B}(X)$ represents a strongly invariant family of projections for C and $\Psi : \Delta \times \Theta \to \mathcal{B}(X)$ is given by Proposition 1.1.

Proposition 2.1. *The mapping $L : T \times \Gamma \to \mathbb{R}_+$ is (h,k)-Lyapunov functional for the pair (C,P) if and only if there exist $l_1, l_2 \in \mathbb{R}$, $l_1 < l_2$ with the properties:*

$$(hL_1) \quad \int_{s}^{t} \frac{||\Phi(\tau,t_0,\theta)P(t_0,\theta)x||}{h(\tau)^{l_1}} d\tau \leq \frac{L(s,s,t_0,\theta,P(t_0,\theta)x) - L(t,s,t_0,\theta,P(t_0,\theta)x)}{h(s)^{l_1}};$$

$$(kL_1') \quad \int_{s}^{t} \frac{||\Psi(t,\tau,\varphi(\tau,t_0,\theta))Q(t,\varphi(t,t_0,\theta))x||}{k(\tau)^{l_2}} d\tau \leq$$

$$\leq \frac{L(t,t,t_0,\theta,\Psi(t,t_0,\theta)Q(t,\varphi(t,t_0,\theta))x) - L(t,s,t_0,\theta,\Psi(t,t_0,\theta)Q(t,\varphi(t,t_0,\theta))x)}{k(t)^{l_2}},$$

for all $(t,s,t_0,\theta,x) \in T \times \Gamma$.

Proof. It is sufficient to justify the equivalence $(kL_1) \Leftrightarrow (kL_1')$ and we use the relations from Proposition 1.1.
If (kL_1) holds, then for all $(t,s,t_0,\theta,x) \in T \times \Gamma$ we have:

$$\int_{s}^{t} \frac{||\Psi(t,\tau,\varphi(\tau,t_0,\theta))Q(t,\varphi(t,t_0,\theta))x||}{k(\tau)^{l_2}} d\tau =$$

$$= \int_{s}^{t} \frac{||Q(\tau,\varphi(\tau,t_0,\theta))\Psi(t,\tau,\varphi(\tau,t_0,\theta))Q(t,\varphi(t,t_0,\theta))x||}{k(\tau)^{l_2}} d\tau =$$

$$= \int_s^t \frac{||\Phi(\tau,t_0,\theta)\Psi(\tau,t_0,\theta)Q(\tau,\varphi(\tau,t_0,\theta))\Psi(t,\tau,\varphi(\tau,t_0,\theta))Q(t,\varphi(t,t_0,\theta))x||}{k(\tau)^{l_2}}d\tau =$$

$$= \int_s^t \frac{||\Phi(\tau,t_0,\theta)\Psi(t,t_0,\theta)Q(t,\varphi(t,t_0,\theta))x||}{k(\tau)^{l_2}}d\tau =$$

$$= \int_s^t \frac{||\Phi(\tau,t_0,\theta)Q(t_0,\theta)\Psi(t,t_0,\theta)Q(t,\varphi(t,t_0,\theta))x||}{k(\tau)^{l_2}}d\tau \leq$$

$$\leq \frac{L(t,t,t_0,\theta,Q(t_0,\theta)\Psi(t,t_0,\theta)Q(t,\varphi(t,t_0,\theta))x)}{k(t)^{l_2}}-$$

$$-\frac{L(t,s,t_0,\theta,Q(t_0,\theta)\Psi(t,t_0,\theta)Q(t,\varphi(t,t_0,\theta))x)}{k(t)^{l_2}} =$$

$$= \frac{L(t,t,t_0,\theta,\Psi(t,t_0,\theta)Q(t,\varphi(t,t_0,\theta))x) - L(t,s,t_0,\theta,\Psi(t,t_0,\theta)Q(t,\varphi(t,t_0,\theta))x)}{k(t)^{l_2}}.$$

Conversely, if (kL_1') is satisfied, then

$$\int_s^t \frac{||\Phi(\tau,t_0,\theta)Q(t_0,\theta)x||}{k(\tau)^{l_2}}d\tau = \int_s^t \frac{||Q(\tau,\varphi(\tau,t_0,\theta))\Phi(\tau,t_0,\theta)Q(t_0,\theta)x||}{k(\tau)^{l_2}}d\tau =$$

$$= \int_s^t \frac{||\Psi(t,\tau,\varphi(\tau,t_0,\theta))\Phi(t,\tau,\varphi(\tau,t_0,\theta))Q(\tau,\varphi(\tau,t_0,\theta))\Phi(\tau,t_0,\theta)Q(t_0,\theta)x||}{k(\tau)^{l_2}}d\tau =$$

$$= \int_s^t \frac{||\Psi(t,\tau,\varphi(\tau,t_0,\theta))Q(t,\varphi(t,t_0,\theta))\Phi(t,t_0,\theta)Q(t_0,\theta)x||}{k(\tau)^{l_2}}d\tau \leq$$

$$\leq \frac{L(t,t,t_0,\theta,\Psi(t,t_0,\theta)Q(t,\varphi(t,t_0,\theta))\Phi(t,t_0,\theta)Q(t_0,\theta)x)}{k(t)^{l_2}}-$$

$$-\frac{L(t,s,t_0,\theta,\Psi(t,t_0,\theta)Q(t,\varphi(t,t_0,\theta))\Phi(t,t_0,\theta)Q(t_0,\theta)x)}{k(t)^{l_2}} =$$

$$= \frac{L(t,t,t_0,\theta,Q(t_0,\theta)x) - L(t,s,t_0,\theta,Q(t_0,\theta)x)}{k(t)^{l_2}},$$

for all $(t,s,t_0,\theta,x) \in T \times \Gamma$. □

Theorem 2.2. *Let (C,P) be a pair with (h,k)-growth, $h\in\mathcal{H}_1\cap\mathcal{H}$ and $k\in\mathcal{K}_1\cap\mathcal{H}$. Then (C,P) has (h,k)-splitting if and only if there exist $L:T\times\Gamma\to\mathbb{R}_+$ a (h,k)-Lyapunov functional for (C,P) and a nondecreasing mapping $\lambda:\mathbb{R}_+\to[1,+\infty)$ such that:*

(L_1) $L(s,s,t_0,\theta,P(t_0,\theta)x)\le\lambda(s)||\Phi(s,t_0,\theta)P(t_0,\theta)x||;$
(L_2') $L(t,t,t_0,\theta,\Psi(t,t_0,\theta)Q(t,\varphi(t,t_0,\theta))x)\le\lambda(t)||Q(t,\varphi(t,t_0,\theta))x||,$

for all $(t,s,t_0,\theta,x)\in T\times\Gamma$.

Proof. We show the equivalence between the conditions (L_2) and (L_2'), using Proposition 1.1.
If (L_2) is verified, then we deduce:

$$L(t,t,t_0,\theta,\Psi(t,t_0,\theta)Q(t,\varphi(t,t_0,\theta))x)=$$

$$=L(t,t,t_0,\theta,Q(t_0,\theta)\Psi(t,t_0,\theta)Q(t,\varphi(t,t_0,\theta))x)\le$$

$$\le\lambda(t)||\Phi(t,t_0,\theta)Q(t_0,\theta)\Psi(t,t_0,\theta)Q(t,\varphi(t,t_0,\theta))x||=$$

$$=\lambda(t)||Q(t,\varphi(t,t_0,\theta))x||,$$

for all $(t,t_0,\theta,x)\in\Delta\times\Gamma$.
In a similar manner, if (L_2') holds, then we obtain:

$$L(t,t,t_0,\theta,Q(t_0,\theta)x)=L(t,t,t_0,\theta,\Psi(t,t_0,\theta)\Phi(t,t_0,\theta)Q(t_0,\theta)x)=$$

$$=L(t,t,t_0,\theta,\Psi(t,t_0,\theta)Q(t,\varphi(t,t_0,\theta))\Phi(t,t_0,\theta)Q(t_0,\theta)x)\le$$

$$\le\lambda(t)||Q(t,\varphi(t,t_0,\theta))\Phi(t,t_0,\theta)Q(t_0,\theta)x||=\lambda(t)||\Phi(t,t_0,\theta)Q(t_0,\theta)x||,$$

for all $(t,s,t_0,\theta,x)\in T\times\Gamma$. □

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