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TRANSIENTS IN HYDRODYNAMIC CONVERTERS - MATHEMATICAL MODELING OF DRIVEN MECHANISM

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Abstract

Hydrodynamic converters are an important part of automatic transmission. Starting an electric motor coupled to a hydrodynamic converter represents a transient regime which begins with the startup of electrical motor drive and last to nominal regime. Knowing this process is important in order to evaluate the parameters used in choosing the electrical drive and the rest of transmission components. This includes the calculus of starting characteristic times of electric drive (which must be short enough) and the exit to an execution mechanism (which must be long enough), determinations of the mechanic load of the drive mechanism, and so on. [1]

1 Introduction

The driving mechanism, according to figure 1, is composed from an electric drive (1), a hydrodynamic transmission (4) and a brake (7) (which simulates the execution element). The electric drive is coupled to the transmission pump, having the same angular velocity, ω_1. The execution element is connected with the turbine part of the transmission, having the same angular velocity ω_2. The input torque is M_1 and the output one M_2. The pump transmit the torque M_{hs} to the turbine. When an hydrodynamic transmission is accelerated (or decelerated) is obtained the condition: M_{hP}=M_{hT}=M_{hs}. This hypothesis is sufficient in the operating area when the transmitted torque is higher than the torque losses.

Characteristic points which defines the starting/stopping moments are M_{hs}=M_{hsz} and ω_{hs}=0 and nominal regimes M_{hs}=M_{hsn}, ω_{hs}=ω_{hsn} can be taken as torque characteristics of the hydrodynamic transmission. The driven mechanism has two characteristic points: starting/stopping regime (ω_{ru}=0; M_{ru}=M_{ruz}) and nominal regime (ω_{ru}=ω_{run}; M_{ru}=M_{run}).

At the initial moment, angular velocities (at the electric drive shaft and at the turbine shaft) are equals to zero, the driving torque is equal to the starting torque (M_m=M_{mz}) and the output at the shaft of the turbine resets (ω_2=0, M_2=0).

[1]MSC (2010): 34A34, 37N05, 80A10

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A part of the driving torque, M_m, is necessary to accelerate the driving shaft, the pump and the corresponding part of the fluid from the transmission, as the torque M_{hs}. The value of M_{hs} depends of the angular velocity's at the input, ω_2, and at the output, ω_2. If those angular velocity's are equals to zero, then the torque of the transmission will be also equal to zero. With the grows of the input rotational velocity, at the output shaft appears a torque different of zero.

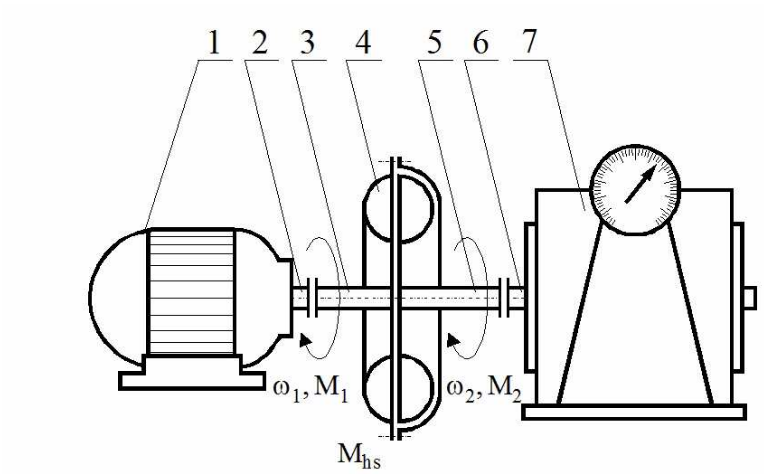

Figure 1: Hydrodynamic transmission sketch

As long as the value of stopping torques M_{hsz} is smaller then the initial torque of the execution mechanism, M_{ruz}, the output shaft will not rotate. After the torque M_{hs} becomes greater then the execution shaft torque, M_{ru}, the output shaft will begun to rotate. The mechanical torque at the moment of the start of rotation of the output shaft is defines by ω_2=0, M_{hs}=M_{ruz}, ω_1=$\omega_1{}^*$ and M_m=$M_m{}^*$. Until that value of the torque, both shafts rotates to the end of the starting process, until the angular velocity's becomes stable. This steady regime is definite as follows: ω=ω_1=ω_{mn}, M_m=M_{mn}, M_{hs}=M_1=M_2, ω_{hs}=ω_2=ω_{hsn}, ω_{ru}=ω_2=ω_{run} and M_{ru}=M_{run}. Presumably that the execution system elements are well chosen, so that at steady regime all parameters are at nominal values.

Symbols:

Basic symbols :			Indices :	
J	kg m	- masic inertia momentum	1	- shaft 1, input of hydrodynamic transmission
I	A	- current	2	- shaft 2, output of hydrodynamic transmission
M	N m	- torque	h	- hydraulic
P	W	- power	hs	- hydrodynamic drive
ΔP	W	- lost power	i, j	- voltage values
t	s	- time	m	- motor
Δt_z	S	- starting interval	max	- maximum value
U	V	- voltage	n	- nominal value
ω	s^{-1}	- angular velocity	P	- pump rotor
ε	s^{-2}	- angular acceleration	pm	- driving motor
f	Hz	- frequency	ru	- executioner devise
			T	- turbine rotor
			z	- stopping (when ω_2=0)

2 Mathematical modeling of driven mechanism

The described mechanism is presented in figure 2, as two shafts with reduced masses. The shaft 1 has the rotational velocity ω_1, the shaft 2 with the rotational velocity ω_1, where $\omega_1 > \omega_2$.
The rotating masses in rotational moving of the electric drive, pump and those of the working liquid are reduced to the shaft 1. The masses of the output shaft, of the turbine and the corresponding part of the working fluid and also the rotating parts of the execution mechanism are reduced to the shaft 2. The elastic characteristic of other moving elements are neglected.
The torque of the driving motor, M_m, act at the left end of shaft 1. This torque is transmitted to the shaft 2. At the right end of shaft 2 act the torque M_{ru}. That torque act against the torque of the transmission. For shafts 1 and 2 can be write the following equations:

$$M_m - M_{i1} = M_{hs} \tag{1}$$

$$M_{hs} - M_{i2} = M_{ru} \tag{2}$$

The inertial torque for both shafts will be:

$$M_{i2} = I_1\dot{\omega}_1 = I_1 = I_1\frac{\partial \omega_1}{\partial t} \tag{3}$$

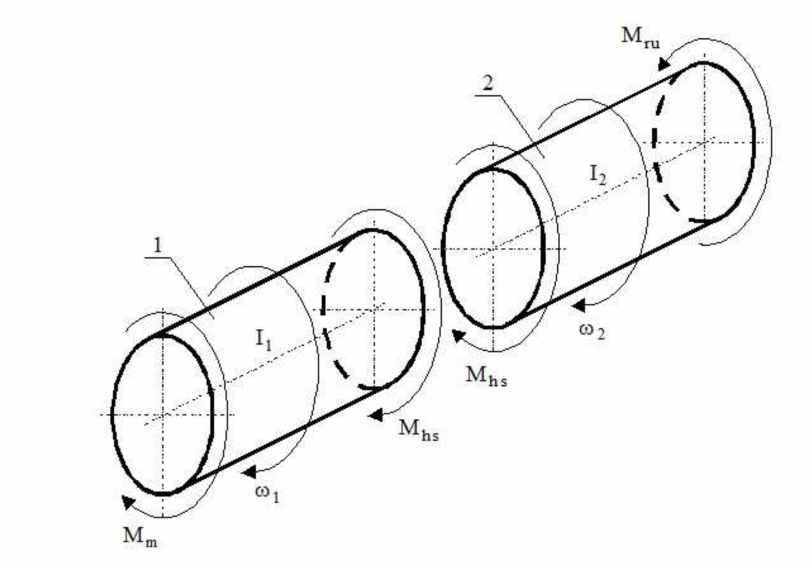

Figure 2: Mathematic model of the driving element

$$M_{i2} = I_2 \dot{\omega}_2 = I_2 \frac{\partial \omega_2}{\partial t} \tag{4}$$

Replacing equations (1) and (2) in (3) and (4), will obtain:

$$M_m - I_1 \frac{\partial \omega_1}{\partial t} = M_{hs} \tag{5}$$

$$M_{hs} - I_2 \frac{\partial \omega_2}{\partial t} = M_{ru} \tag{6}$$

Equations (5) and (6) can be solved graphic; the torque characteristics of the driving motor and of the transmission are given in figure 3, from left to the right. The torque characteristic of the transmission - M_{hsz} – is added to the torque characteristic of the electric drive. This characteristic shows the relation between the torque M_{hs} when the rotational velocity of tye output, ω_2, is zero, when the ratio between the two rotational velocities are zero (ω_2/ω_1=0).
This point, having the coordinates ($\omega_1{}^*$, $M_{hsz}{}^*$) can be easily view on the characteristic M_{hsz}. This point defines the working point when the output torque is equal to the execution mechanism torque. Here is obtained ω_1=$\omega_1{}^*$, M_{hs}=$M_{hsz}{}^*$=M_{ruz}, ω_2=0. Until this moment, only the shaft is in rotational movement, so only equation (5) must be solved. For solving equation (5), is supposed that the starting regime is divided in two time intervals with constant rotational velocity, driving torque and transmission torque, according to the last squares method. The parameters being constants along each time interval, the number of intervals should be long enough.

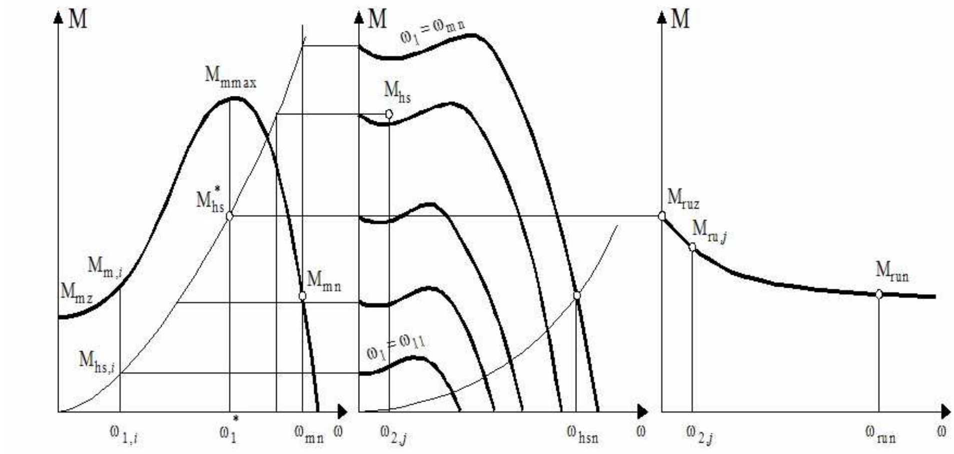

Figure 3: Graphic solving of the torque equations

According to that assumption, the differential equation (5) becomes an ordinary linear equation:

$$M_m - I_1 \frac{\Delta\omega_1}{\Delta t} = M_{hs} \tag{7}$$

Equation (7) is solved along Δt for known variations $\Delta\omega_1$. For an "i" time interval in which the angular velocity grows from $\omega_{1,i-1}$ to $\omega_{1,i}$ will have:

$$\Delta t_i = \frac{\omega_{1,i} - \omega_{1,i-1}}{M_{m,i-1} - M_{hs,i-1}} I_1 \tag{8}$$

When all time intervals and angular velocity increments are known, it is possible to evaluate the angular acceleration in each time interval:

$$\varepsilon_{1,\mathrm{i}} = \frac{\omega_{1,\mathrm{i}} - \omega_{1,\mathrm{i}-1}}{\Delta \mathrm{t_i}} \tag{9}$$

The starting period of shaft 1 (of the electric drive) until the angular velocity is obtained by summarizing the time intervals:

$$\Delta t* = \sum_{i=1}^{i=i*} \Delta t_i \tag{10}$$

From the moment when the torque $M_{hs}=M_{ruz}$ appears to the output shaft of the transmission and the shaft 1 touch the angular velocity $\omega_1=\omega_1{}^*$, the shaft 2 begins to rotate. At that moment, both shafts accelerate and the starting period is finished when is obtained the angular velocity ω_1=ω_{mn}. The starting period is also divided in time intervals having constant angular acceleration, torque to the driving motor and torque to the execution mechanism. In figure 3 are presented only two time intervals.

Because of presented hypothesis, the differential equations (5) and (6) becomes ordinary linear equations, as (7) and

$$M_{hs} - I_2 \frac{\Delta\omega_2}{\Delta t} = M_{ru} \tag{11}$$

Equations (7) and (11) are solved as follows. First, the value of Δt is found for a value of $\Delta\omega_1$ known from (7) and (8). For a Δt known, the value of $\Delta\omega_2$ is found from (11) and (12), using the least square method:

$$\Delta t^{**} = \sum_j \Delta t_j \tag{12}$$

Now, the values of M_m, M_{hs} and M_{ru} can be calculated for known values $\Delta\omega_1$ and $\Delta\omega_2$, ω_1 and ω_2. These values are used in the next iteration.
The described procedure is continued until is obtained a stationary value for the angular velocity of the execution element. Generally, the procedure is finished when the iteration error is less then ±5%.
The starting time period for second shaft, for execution element, is the sum of time intervals:

$$\Delta\omega_{2,j} = \frac{M_{hs,i*+j-1} - M_{ru,i*+j-1}}{I_2} \Delta_{i*+j} \tag{13}$$

3 Solution analyze

This calculation can be used to determine the starting time period both for a driven mechanism and for finding the addiction between the input and output torque, angular velocity and power losses during the acceleration period.
The starting period for a driving motor is:

$$\Delta \mathrm{t_{zpm}} = \Delta\ \mathrm{t}^* + \Delta\ \mathrm{t}^{**} \tag{14}$$

The starting period of an execution mechanism is:

$$\Delta \mathrm{t_{zm}} = \Delta\ \mathrm{t}^{**} \tag{15}$$

The addiction between the input and output torque, angular velocity and power losses during the acceleration period are presented in figure 4.
The power losses in hydrodynamic transmission are:

$$\Delta P_{hs} = P_1 - P_2 = M_1 \cdot \omega_1 - M_2 \cdot \omega_2 \quad (16)$$

Analyzing figure 4, results that the load of electric motor and of hydrodynamic transmission is obtained in starting period. That's why for a specific electric motor a hydrodynamic transmission must be chosen so that the motor starts as quick as possible. More, the transmission must also assure the start of the execution element. This starting period should be long enough in order to avoid excessive load of execution element and enough short in order to assure a rapid start.

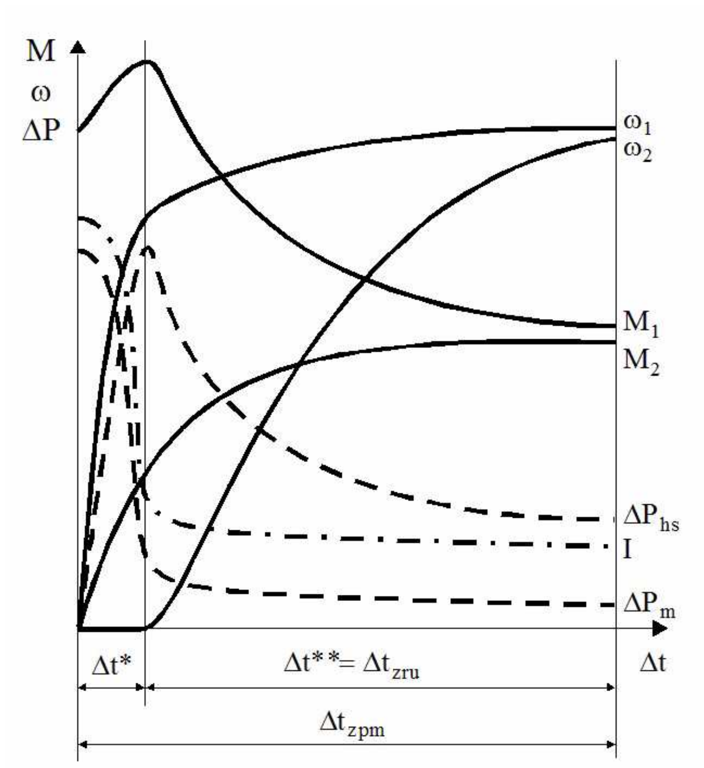

Figure 4: The addiction between the input and output torque, angular velocity and power losses during the starting period of a hydrodynamic transmission

4 The behavior of hydrodynamic transmissions in transients

Starting from the momentum equation for each component of a hydrodynamic transmission, will have:

- for pump

$$M_p = \rho\dot{Q}\dot{(}r_{2P}\dot{V}_{u2P} - r_{1P}\dot{V} - u1P) \tag{17}$$

- for turbine

$$\mathrm{M_T} = \rho \cdot \mathrm{Q} \cdot (\mathrm{r_{1T}} \cdot \mathrm{v_{u1T}} - \mathrm{r_{2T}} \cdot \mathrm{v_{u2T}})\,, \tag{18}$$

- for reactor

$$\mathrm{M_R} = \rho \cdot \mathrm{Q} \cdot (\mathrm{r_{1R}} \cdot \mathrm{v_{t1R}} - \mathrm{r_{2R}} \cdot \mathrm{v_{t2R}})\,, \tag{19}$$

and, using classical notations form turbo machines theory, will obtain:

$$\omega_\mathrm{P} \cdot \left(\left(\mathrm{r_{2P}^2} - \mathrm{r_{1P}^2}\right) - \left(\mathrm{r_{1T}^2} - \mathrm{r_{2T}^2}\right) \cdot \frac{1}{\mathrm{i}} \right) = (\mathrm{K_P} - \mathrm{K_R} - \mathrm{K_T}) \cdot \mathrm{Q} \tag{20}$$

where "i" is the gear ratio ($\mathrm{i} = \omega_P/\omega_T$) and, as input values are the rotational velocity at primary machine shaft (pump), and as output value – flow in the transmission. Also, can be noticed that in the left side of equal sing there are exclusively dependent by transmission geometry.

In order to analyze in time domain, in relation (20) will consider:

$$\omega = \Omega + \overline{\omega} \cdot \exp\left(2 \cdot \pi \cdot f_\omega \cdot t \cdot j\right) \tag{21}$$

$$Q = \overline{Q} + q \cdot \exp\left((2 \cdot \pi \cdot f_Q \cdot t + \delta_Q) \cdot j\right) \tag{22}$$

where: Ω=nominal value of angular velocity in transients, $\overline{\omega}$=pulsation rate of the angular velocity, f_ω=frequency of oscillation of angular velocity, t=time, $\overline{Q}$=nominal value of theoretic flow in transmission, q = flow pulsation, f_Q = frequency of oscillation of flow, δ_Q=the phase difference between the occurrence of the angular velocity signal and the signal flow, j=complex variable ($j=(-1)^{1/2}$).

Numerical values where determinate from experiments as follows: Ω=100,531 rad/sec, $\overline{\omega}=(0\div10\%)\Omega$, $\mathrm{f}_\omega=(0\div10)$Hz, $\overline{Q}$=0,17752 $\mathrm{m^3/s}$, $\mathrm{q}=(0 \div 10\ \%)\overline{Q}$, $\mathrm{f}_Q=(0\div10)$Hz, $\delta_Q=(0\div\pi/2)$ Hz .

Using the above values, were obtained following results:

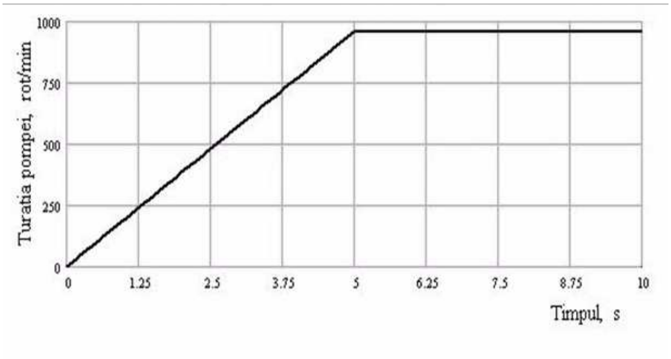

Figure 5. Speed variation in time, at starting

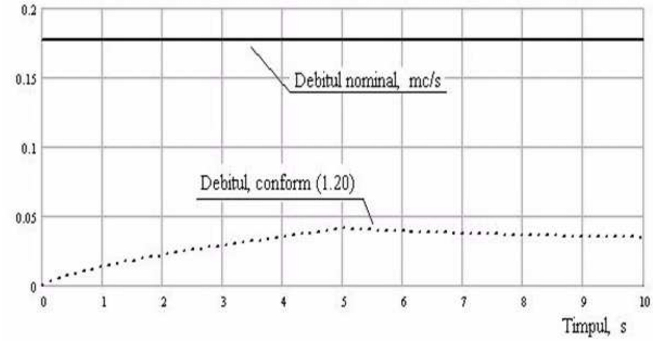

Figure 6. Flow variation in time, correlated with speed

For the transfer function angular velocity – theoretic flow was determinate BODE diagram, as presented in figures 7 – 13, for several values of speed.

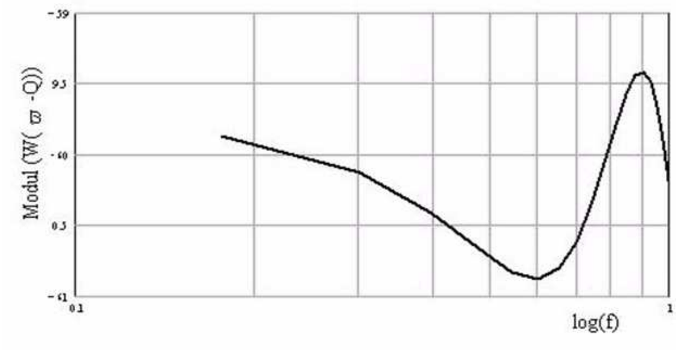

Figure 7. t = 0,1 s, n = 19,2 rot/min

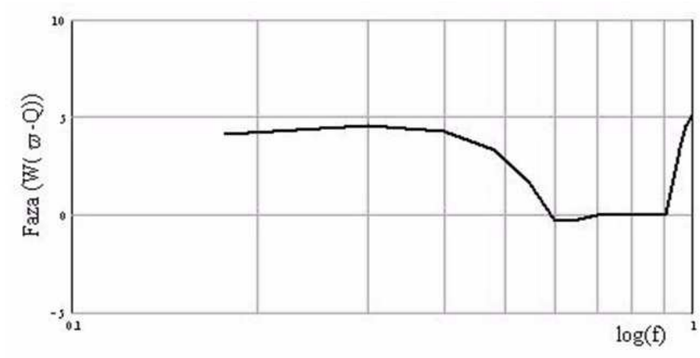

Figure 8. t = 0,1 s, n = 19,2 rot/min

Figure 9. t = 2,5 s, n = 480 rot/min

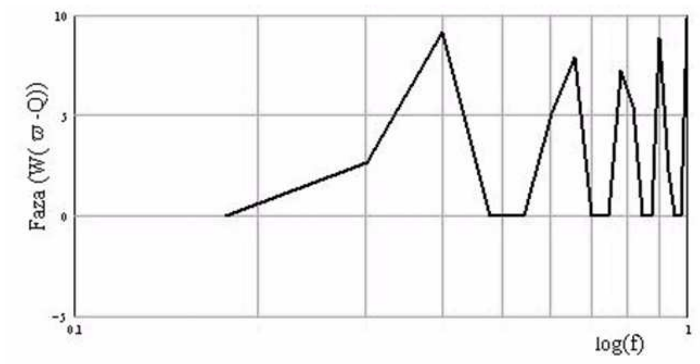

Figure 10. t = 2,5 s, n = 480 rot/min

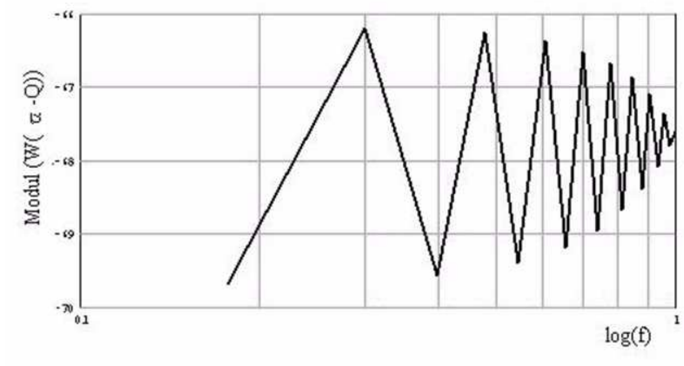

Figure 11. t = 5 s, n = 960 rot/min

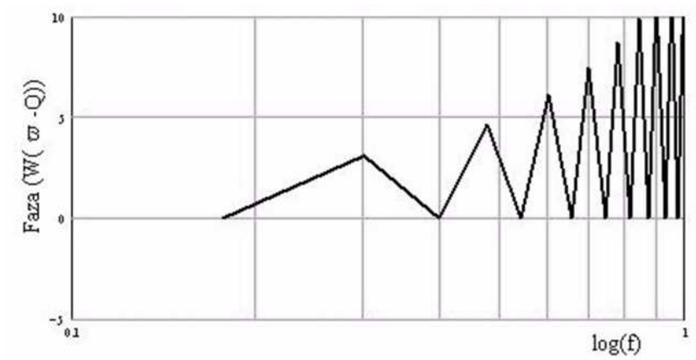

Figure 12. t = 5 s, n = 960 rot/min

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