An Effective Exploitation of Volterra Filter for Denoising MRI Images

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Abstract

Image denoising is most effective for achieving both noise reduction and feature preservation. To recover the original image various noise removal techniques such as, linear minimum mean squared error method (LMMSE), histogram based denoising, wiener filter and maximum likelihood (ML) approach are used. The main problem in these filter is resulting images are often blurred and causes spatial flattering. In this paper, Volterra filter is proposed to eliminate the noise to the maximum extent, without altering the quality of an original MRI image. Among all the denoising filters, Volterra shows its excellence with the highest peak signal to noise ratio (PSNR) value and the lowest mean square error value (MSE). The performance is evaluated to validate and estimate the performance of visual quality of an image.

Keywords- Magnetic resonance imaging, Volterra filter, peak signal to noise ratio, linear minimum mean squared error method, histogram based denoising, wiener filter and maximum likelihood approach.

I INTRODUCTION

Image Restoration is the operation of taking a corrupt/noisy image and estimating the clean, original image. Corruption may come in many forms such as motion blur, noise and camera mis-focus.[1] Image restoration is performed by reversing the process that blurred the image and such is performed by imaging a point source and use the point source image, which is called the Point Spread Function (PSF) to restore the image information lost to the blurring process. Image restoration is different from image enhancement in that the latter is designed to emphasize features of the image that make the image more pleasing to the observer, but not necessarily to produce realistic data from a scientific point of view. Image enhancement techniques (like contrast stretching or de-blurring by a nearest neighbor procedure) provided by imaging packages use no a priori model of the process that created the image.

With image enhancement [2] noise can effectively be removed by sacrificing some resolution, but this is not acceptable in many applications. In a fluorescence microscope, resolution in the z-direction is bad as it is. More advanced image processing techniques must be applied to recover the object. The objective of image restoration techniques is to reduce noise and recover resolution loss. Image processing techniques [3] are performed either in the image domain or the frequency domain. The most straightforward and a conventional technique for image restoration is deconvolution, which is performed in the frequency domain and after computing the Fourier transform of both the image and the PSF and undo the resolution loss caused by the blurring factors. This deconvolution technique, because of its direct inversion of the PSF which typically has poor matrix condition number, amplifies noise and creates an imperfect deblurred image. Also, conventionally the blurring process is assumed to be shift-invariant. Hence more sophisticated techniques, such as regularized deblurring, have been developed to offer robust recovery under different types of noises and blurring functions. Image denoising [4] is an important problem in image processing since noise may interfere with visual or automatic interpretation.It is the fundamental challenges in the field of image processing and is important in a wide variety of applications such as object recognition, photo enhancement, and image restoration[5]. Image denoising may be defined as the process of recovering the original image from a noisy or degraded image by using a priori knowledge of the degradation phenomenon. Hence, restoration algorithms are based on modeling the degradation and applying the inverse process to recover the original image. With the advent of imaging technology, medical science has obtained its enhanced insight with various imaging modalities, which are able to capture the functionalities of inner body. This ranges from Xray, MRI, CT, PET and the list goes on [6]..

The most common phase in which, an image is caught by noise is while acquiring or transferring or compressing an image. Presence of noise renders poor quality to an image and here comes the necessity to eliminate noise and to have an enhanced image. Image denoising is the methodology that aims at eliminating the noise from an image. A denoising technique saves the signal which is corrupted by noise.MRI plays a crucial role in neuroscience and medical diagnosis.MRI images are always corrupted with noise. Removing noise from images is crucial but it is not an easy task. Filtering algorithm is the most common method used to remove noise.Medical image processing relies on denoising techniques, so as to diagnose the disease or to find the severity of the disease perfectly. Perfect analysis of medical images depends on effective noise removal which is an important step of image enhancement. All the image enhancement techniques support at segmentation and classification, which is classifying between the normal and the affected area. The so-produced enhanced images well suits a physician to diagnose or to compute the degree of severity of the disease. The goodness of a denoising methodology lies on the preservation of edges and the degree of noise that has been removed. The performance of denoising methods is measured by PSNR and MSE.

Wiener filter (Linear Minimum Mean Squared Error) purpose [7] is to reduce the amount of a noise in a signal. This is done by comparing the received signal with a estimation of a desired noiseless signal. Wiener filter is not an adaptive filter as it assumes input to be stationery.

It takes a statistical approach to solve its goal, goal of the filter is to remove the noise from a signal. Before implementation of the filter it is assumed that the user knows the spectral properties of the original signal and noise. Spectral properties like the power functions for both the original signal and noise. And the resultant signal required is as close to the original signal. Signal and noise are both linear stochastic processes with known spectral properties. The aim of the process is to have minimum mean- square error. That is, the difference between the original signal and the new signal should be as less as possible. The main drawback in this filter is if you don't get much change in noise or signal a fixed filter will be less expensive computationally. It will also be immune to estimation variance, whenever you estimate a stochastic parameter you will add noise to the system.

Maximum likelihood filter[8] will overcome the drawback of wiener filter, consider image denoising as an estimation of the “true” image from noisy data. The images are considered to be defined over a discrete regular grid and we denote by a pixel value at site. We consider an uncorrelated noise model defined by a parametric noise distribution (namely the likelihood), with a space-varying unknown parameter. Then, denoising an image is assumed to be equivalent to find the best estimate of. At each site, the MLE defines an estimate of the underlying parameter from a set. A drawback of the filter is the suppression of thin and dark details in the regularized images.

The histogram [9] will overcome the drawback of maximum likelihood filter, here the image is first represented as histogram, and then fuzzy rule are applied on this. The images taken up for the experimental analyses is subjected to the fuzzy based filter for SP noise removal. The proposed algorithm exhibits superiority over traditional algorithms and recently proposed ones in terms of visual quality, peak signal to noise ratio and mean square error. When an image is interfered by salt and pepper noise, there will appear some dark spots and bright spots on the image, which seriously affect image’s quality. Thus filtering out these spots ton get a clear image is very important task in image processing. Although other filters are also proposed for removing such type of noise, but somehow they suffer from inability to remove those dark spots and bright spots on the image simultaneously. Hence there feasibility is poor. So here a fuzzy based histogram adaptive filter [10] is introduced, which performs fuzzy processing to the histogram of a original image. The advantage of using this type of filter is that, it utilizes information of original image’s histogram and its startup time is shorter. The histogram provides a convenient summary of the intensities in an image, but is unable to convey any information regarding spatial relationships between pixels.

Adaptive non-local means filter is used to deal with magnetic resonance images (MRI) with spatially varying noise levels (for both Gaussian and Rician distributed noise).Most filtering techniques assume an equal noise distribution across the image. When this assumption is not met, the resulting filtering becomes suboptimal. This is the case of MR images with spatially varying noise levels, such as those obtained by parallel imaging (sensitivity-encoded), intensity inhomogeneity-corrected images, or surface coil-based acquisitions. We propose a new method where information regarding the local image noise level is used to adjust the amount of denoising strength of the filter. Such information is automatically obtained from the images using a new local noise estimation method.The main drawback is some edges will be blurred and do not perform sharpness.

Denoising MRI images is an important preprocessing step required in many of the automatic computed aided-diagnosis systems in neuroscience. Rician noise occurs in the MRI image during acquisition. Non local mean filter is used for denoising. But the parameter selection is not optimized. The proposed method removal of rician noise in MRI images using bilateral filter by fuzzy trapezoidal membership function improves the denoising efficiency at various noise variances, preserves the fine structures and edges. The fuzzy weights were obtained with the statistical features such as local mean (μi) and global mean (μg) by constructing trapezoidal membership function. Bilateral filter is used to preserve the edges by smoothening the noises in MRI image and preserves the structural information. Local filter preserves the edges. MRI images are restored by multiplying its corresponding fuzzy weight with the restored image of local order filter and bilateral filter but leads to gradient distortion.

To remove noise self-similarity and soft shrinkage is important [11, 15]. For that brightness of the neighboring pixels are also important [12]. To suppress various noises at different noise levels denoising is also necessary [13]. Only certain patches are blurred out during denoising of signal [14].To get the possible image information package of the image is used [16] to reconstruct the noisy pixels [17] of the noisy file set [18].

Image denoising based on e-active filtering in 3D transform domain by combining sliding-window transform processing with block-matching. We process blocks within the image in a sliding manner and utilize the block-matching concept by searching for blocks which are similar to the currently processed one. The matched blocks are stacked together to form a 3D array and due to the similarity between them, the data in the array exhibit high level of correlation. We exploit this correlation by applying a 3D decorrelating unitary transform and electively attenuate the noise by shrinkage of the transform coefficients.The subsequent inverse 3D transform yields estimates of all matched blocks. After repeating this procedure for all image blocks in sliding manner, the final estimate is computed as weighed average of all overlapping block estimates.The main disadvantages are computationally intensive for large frame sizes.

In this work, Volterra filters is proposed to optimize the trade-off between noise removal and edge preservation. We denoise the MRI images by employing Volterra filter. In the given image, type of noise will be rician noise. Apply the created Volterra filter on to the noisy image with the neighbour window size of 3X3 and thus denoised image is obtained. Also, comparison is made between Volterra filter and others such as Linear Minimum Mean Squared Error (Wiener filter), Histogram based approach and maximum likelihood. Experimental results show that the Volterra filter outperforms the other ways of de noising and is shown via PSNR and MSE values. Volterra filter has got the highest PSNR and the lowest MSE values.

Significant contribution of the paper:

1) Image denoising is most effective for achieving both noise reduction and feature preservation.

2)Volterra filter is proposed to eliminate the noise to the maximum extent, without altering the quality of an original MRI image.

3) Thus, combined results show the PSNR value and the lowest MSE.

The manuscript is organized as follows:- section II organised with Review of literature . Section III contains the results and performance analysis and finally concluding remarks is presented.

II REVIEW OF LITERATURE

Different methodologies for noise reduction giving an insight as to which algorithm should be used to find the most reliable estimate of the original image data given its degraded version.

Sashikant Agrawal et al. [19] have proposed a medical image denoising algorithm by using Discrete Wavelet Transform (DWT). This work compares the efficiency of the wavelet based thresholding technique at various levels and degrees of random noise. The performance of the thresholding technique is analyzed for wavelet family Haar, db2, db4, sym2, sym4, bior1.1and bior1.3. Out of these, Haar (db1) wavelet has performed well when compared with others and db4 wavelet hasshown the poorest performance with the least PSNR and the highest MSE values.

Kanwaljot Singh Sidhu et.al [20] claims that db3 wavelet proves its efficiency over Haar wavelet. This is because it removes certain level of speckle noise in the medical images. Also, this enhances the visual quality of the medical images. In this work, denoising is carried out at both soft and hard threshold values. Sashikant et.al [21] have proposed well with MR images, which are corrupted by random noise. This algorithm yields better results both in terms of visual quality and Mean Square Error values.

Paul Scheunders and Steve De Backer et.al [22] have presented a Bayesian wavelet based denoising procedure for multi-component images. The proposed procedure fully accounts for the multi-component image co-variances, makes use of Gaussian scale mixtures as prior models that approximate the marginal distributions of the wavelet coefficients well and makes use of a noise free image as extra prior information. Also, it is given that all such prior information is available with specific multicomponent image data.

Paul Bao and Lei Zhang et.al [23] have proposed an MRI image denoising scheme using an adaptive wavelet thresholding technique. The proposed scheme multiplies the adjacent wavelet subbands to amplify the significant features and then the thresholding is applied to multiscale products for better differentiating the edge structures from noise. This adaptive threshold was formulated to remove most of the noise.

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III PROPOSED WORK

This paper, proposes a Volterra filter for removing noise from MRI images. Apply the created Volterra filter on to the noisy image, type of noise will be Rician noise with the neighbour window size of 3X3 and thus denoised image is obtained. Thus, the implementation result shows that the proposed algorithm provides better signal to noise ratio MSR and improves the PSNR.

The Volterra filter is created by using

$$y(i,j) == \sum_{i=1}^{9} h(i)u(i) + d\sum_{i=1}^{9}\sum_{j=1}^{9} \Psi(i,j)u(i)u(j) \qquad (1)$$

The linear coefficients h (i) of the proposed filter provides powerful noise cancellation in uniform gray zones. Since the sum of the linear coefficients of the filter is equal to one, it acts as a ideal low pass filter resulting in blur. The nonlinear coefficients Ψ (i, j) compensate for the blurring due to the linear term and preserve the edges and high frequency components. The resulting image shows higher quality detail preserved image than that obtained by simple linear filtering.

From Eq. (1) 'd' is a logic variable related to the output of the local decision algorithm. The value of d can be defined as

$$d = \begin{cases} 1 & if\ (\frac{1}{9}\sum_{i=1}^{9} u_i^2 - \bar{u}^2) \\ 0 & \end{cases} \geq t\ , elsewhere \qquad (2)$$

From Eq. (2) 't' is the predefined threshold and it can be determined using the local variance estimator. 'u_i' and '$\overline{u}$' are the input and mean value of the window 'w'. The two conditions which were used to design the filter are given in Eq. (3)

$$\sum_{i=1}^{9} h(i) = 1 \qquad (3)$$

$$\sum_{i=1}^{9}\sum_{j=1}^{9} \Psi(i,j) = 0$$

The Volterra series model is the most widely used model in nonlinear adaptive filtering. The Volterra series expansion can be seen as a Taylor series expansion with memory.

A nonlinear continuous function y=f(x) can be expanded to a Taylor series, at $x=x_0$. is given in Eq. (4)

$$f(x) = \sum_{l=0}^{\infty} \frac{1}{l!}\frac{\partial^l f(x)}{\partial x^l}|_{x=x_0}(x-x_0)^l = \sum_{l=0}^{\infty} a_l(x-x_o)^l \qquad (4)$$

Volterra consists of a non-recursive series in which the output signal is related to the input signal as follows in Eq.(5)

$$\sum_{l_1=0}^{\infty}\sum_{l_2=0}^{\infty}\cdots\sum_{l_p=0}^{\infty} w_{op}(l_1,l_2 \ldots l_p)x(k-l_1)\ldots\ldots x(k-l_p).$$

$$+ y(k) = w_{O0} + \sum_{l_1=0}^{\infty} w_{O1} l_1 x(k-l_1) + + \sum_{l_1=0}^{\infty}\sum_{l_2=0}^{\infty} w_{o2} l_1 l_2 x(k-l_1) l_1 x(k-l_2) \qquad (5)$$

The model is attractive in because the expansion is a linear combination of nonlinear functions of the input signal [10].

The coefficients $w_{op}(l_1, l_2, \dots l_p)$ are the coefficients of a nonlinear combiner based on Volterra series, and called the Volterra series kernels (symmetric). The reason using the symmetric kernels is, it can greatly simplifies the analysis, since the order $\tau's$of becomes unimportant. The result in the expressions are easy to handle and reduces the amount of calculation. The symmetric kernels are shown to be unique and asymmetric kernels are not.

The Volterra series expansion generalizes the Taylor series given in in Eq. (6)

$$y(k) = \sum_{p=0}^{P} \mathcal{H}_p[x(k)] \quad (6)$$

where the terms are given in in Eq. (7)

$$\mathcal{H}_p[x(k)] = \sum_{l_1=0}^{L-1} \sum_{l_2=0}^{L-1} \dots \sum_{l_p=0}^{L-1} w_{op}(l_1, l_2, \dots l_p) x(k-l_1)\, x(k-l_2) \dots x(k-l_p) \quad (7)$$

The truncatedVolterra filter has P[th] nonlinearity order and memory of length L-1 given in integral form Eq. (8). The

$$y(t) = \int_{-\infty}^{\infty} w_{o1}(\tau_1) x(t-\tau c_1) d\tau_1$$
$$+ \int\int_{-\infty}^{\infty} w_{o2}(\tau_1,\tau_2) x(t-\tau_1) x(t-\tau_2) d\tau_1\tau_2$$
$$+ \int\int\int_{-\infty}^{\infty} w_{o3}(\tau_1,\tau_2,\tau_3) x(t-\tau_1) x(k-\tau_2) x(t-\tau_3) d\tau_1\tau_2\tau_3$$
$$+ \int\int \dots \int_{-\infty}^{\infty} w_{op}(\tau_1,\tau_2,\dots\tau_p) x(t-\tau_1) x(t-\tau_2) x(k-) d\tau_1\tau_2 \dots d\tau_p + \cdots \quad (8)$$

Linear combination of nonlinear functions of the input signal, the input-output relationship can be expressed easily in a vector form is given inEq. (9)

$$H_3(w_1,w_2,w_3) = \iint\int_{-\infty}^{\infty} w_{o3}(t_1,t_2,t_3) \exp[-j(w_1t_1 + w_2t_2 + w_3t_3)]\, d\, t_1 dt_2 dt_3 \quad (9)$$

This representation provides a sinusoidal response, which is closely related to the harmonic distortion.

Volterra Filters in Frequency Domain:

Harmonic distortion and inter-modulation products may be expressed in terms of the frequency response.

$$y(t) = \int_{-\infty}^{\infty} h_1(t,\tau_1) x(\tau_1) +$$
$$+ \int\int_{-\infty}^{\infty} h_2(t,\tau_1,\tau_2) x(\tau_1) x(\tau_2) d\tau_1\tau_2 +$$
$$+ \int\int\int_{-\infty}^{\infty} h_3(t,\tau_1,\tau_2,\tau_3) x(\tau_1) x(\tau_2) x(\tau_3) d\tau_1\tau_2\tau_3$$
$$+ .. \quad (10)$$

For a Volterra filter with P=3, the input is given in harmonic distortion and inter-modulationEq. (11)

$$HD_3 = \frac{A^2}{2} \cdot \frac{H_3(w_1,w_2,w_3)}{H_1(w)}$$
$$IM_3 = \frac{3A^2}{2} \cdot \frac{H_3(w_1,w_2,-w_3)}{H_1(w)} \quad (11)$$

The generalization of Volterra filters to the time-varying case is conceptually easy . The time-domain impulse response requires an additional time variable, so $h_1(t,\tau)$ represents the system output at time, if the impulse has been applied at time τ.

Linear combination of nonlinear functions of the input signal, the input-output relationship can be expressed easily in a vector form

$$y(k) = X^T(k) w \quad (12)$$

For a Volterra filter with P=2, the input is

$$X(k) = [1, x(k), \dots . x(k-L+1) | x^2(k), x(k)x(k-1), \dots x^2(k-L+1)]^T \quad (13)$$

and w contains all the kernel coefficients given in in Eq. (14)

$$w = [w_{o0}, w_{o1}(0), \dots w_{o2}(L-1) | w_{o2}(0,0), w_{o2}(1,0), \dots, w_{o2}(L-1, L-1)]^T \quad (14)$$

LMS Volterra filter:

The objective function to be minimized is the Mean Square Error

$$MSE = E[e^2(k)] = E[(d(k) - y(k))^2] \quad (15)$$

The instantaneous squared error

$$e^2(k) = [d(k) - X^T(k) w(k)]^2 \quad (16)$$

is minimized iteratively.

The filtered coefficients are adjusted according to the negative gradient direction

$$w(k+1) = w(k) - \mu \nabla_w e^2(k) \quad (17)$$

The gradient is

$$\nabla_w e^2(k) = \frac{\partial e^2(k)}{\partial w} = 2e(k) \frac{\partial e(k)}{\partial w} = -2e(k) X(k) \quad (18)$$

It is advisable to have different convergence factors for the different kernel or different nonlinearity order.

For the particular case when the order is P=2, we get

$$w(l_1;k+1) = w(l_1;k) + \mu_1 e(k)x(k-l_1)$$
$$w(l_1,l_2;k+1) = w(l_1,l_2;k) + \mu_2 e(k)x(k-l_1)x(k-l_2) \qquad (19)$$

The convergence factors are chosen according to

$$0 < \mu_1,\mu_2 < \frac{1}{trace\{\boldsymbol{R}\}} < \frac{1}{\lambda_{max}} (20)$$

where$\boldsymbol{R} = E[X(k)X^T(k)]$ and $\lambda_{max} = max_i\{eigval_i[\boldsymbol{R}]\}$

The convergence speed depends on the eigenvalue spreading.

LMS Volterra Algorithm:

Initialization:

$$X(0) = W(0) = [0 \ldots\ldots,0]^T \qquad (21)$$

From Eq. (21), For k>0, the instantaneous error computation is given in

$$e(k) = d(k) - X^T(k)w(k) \qquad (22)$$

The coefficient adjustment for a LMS Volterra filter of order P with L-1 delay elements is given Eq. (23).

$$W(k+1)=w(k)+2e(k)diag\{\mu_{1\ldots}\mu_{1\ldots}|\mu_{p\ldots}\mu_{p\ldots}|\}X(k) \ (23)$$

In general, LMS Volterra filter has a slow convergence speed, due to the eigenvalue spread.

RLS Volterra Algorithm:

RLS algorithms achieve fast convergence. The objective function is different from the LMS case by means of $j(k)$

$$= \sum_{i=0}^{k} \lambda^{k-i} e^2(i)$$
$$= \sum_{i=0}^{k} \lambda^{k-i} [d(i) - X^T(i)\, w(k)]^2 \qquad (24)$$

From Eq. (24) the parameter λ controls the memory span of the adaptive filter (0<λ<1). By differentiating this function with respect to the filter coefficients w(k) and setting the derivative to zero. $w(k) = [\sum_{i=0}^{k} \lambda^{k-i} X(i)X^T(i)]^{-1} \sum_{i=0}^{k} \lambda^{k-i} X(i)d(i)$ (25)

The optimal coefficients can be computed as given in Eq. (26).

$$w(k) = \boldsymbol{R}_D^{-1}(k) r_D(k) \qquad (26)$$

If the deterministic correlation matrix of the input vector is denoted by

$$R_D(k)=\sum_{i=0}^{k} \lambda^{k-i} X(i)X^T(i)$$

and

$$R_D(k)=\lambda R_D(k-1)+X^T(k)x(k) \qquad (27)$$

is given Eq. (27).and the deterministic cross correlation vector between the input vector and the desired output is given in Eq. (28).

$$R_D(k)=\sum_{i=0}^{k} \lambda^{k-i} X(i)d^T(i)$$

and

$$R_D(k)=\lambda R_D(k-1)+X^T(k)d(k) \qquad (28)$$

RLS Volterra Algorithm:

Initialization $X(0)=W(0)=[0,,,0]^T$
and $R_D(-1)=\Delta i$
For $k \geq 0$
$E(k)=d(k)-X^T(k)w(k-1)$
$g(k)=\lambda^{-1}R_D^{-1}(k-1)x(k)/1+\lambda^{-1}R_D^{-1}(k-1)x(k)$
$w(k)=w(k-1)+\mu g(k)e(k)$
$R_D(-1)=\lambda^{-1}R_D^{-1}(k-1)x(k)/1+\lambda^{-1}R_D^{-1}(k-1)x(k)$

The following can also be computed if needed.

$$y(k) = w^T(k)x(k)$$
$$e(k)=d(k)-x^T(k)w(k) \qquad (29)$$

From Eq. (21) Thus, the Volterra filter is created and is applied over the noisy image. The steps involved in the proposed method are

Step-1: For a given image, select the type of noise in order to obtain a noisy image. In this work, we employed three noises such as Rician noise, Gaussian noise and Random field noise. In this work, we employed Rician noise, Gaussian noise and Random field noise.

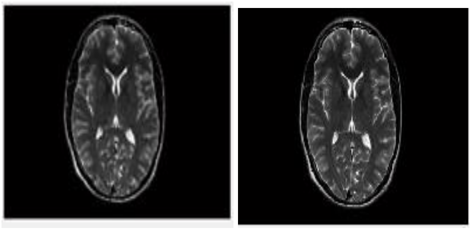
Step-2: Apply the created Volterra filter on to the noisy image with the neighbour window size of 3X3.

Step-3: The denoised image is obtained.

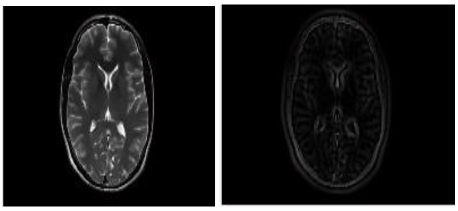
MR images are corrupted by Rician noise, which arises from complex Gaussian noise in the original frequency domain measurements. The Rician probability density function for the corrupted image intensity *x* is given by

$$p(x) = x/\sigma^2 \exp(-x^2+A^2/2\sigma^2) I_0(xA/\sigma^2) (30)$$

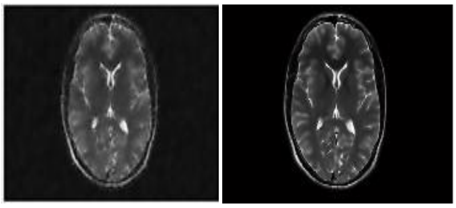
From Eq. (30) where *A* is the underlying true intensity, σ is the standard deviation of the noise, and I_0 is the modified zeroth order Bessel function of the first kind.



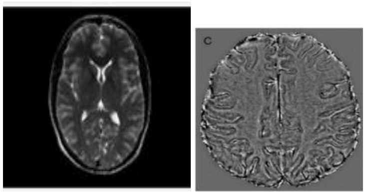
(a) (b)



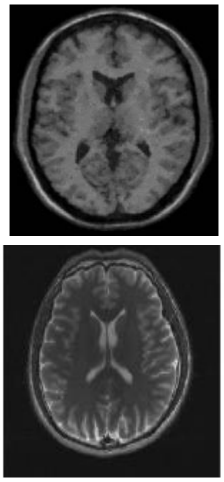
(c) (d)



(e) (f)



(g) (h)



(i) (j)

Fig.1 (a) Original Image (b) Noisy Image (Rician noise) (c) Volterra Filter (d) Linear Minimum Mean Squared Error (e) Histogram based denoising (f) Wiener Filter (g) ML Estimation. (h) Bilateral Filter (i)Adaptive non local mean filter.(j) Block-matching and 3D filtering

IV RESULTS & PERFORMANCE ANALYSIS

In this work, a normal image is denoised with three different types of noises including, Rician noise. All the images are shown in Fig 1,. Among all the techniques that were used to denoise an image, Volterra filter performs well with good visual quality.

Fig. 1(a) is the original images and 1(b) is corrupted images with Rician noise. Figures ranging from 1(c) to (g) are denoised images, which are made out by Volterra filter, Linear Minimum Mean Squared Error method, Histogram based approach, wiener filter and maximum likelihood approach. Volterra filter works well irrespective of the type of noise. The performance is constant without any variation. In order to prove this, two measures are exploited here. They are PSNR and MSE.

A good denoising filter is expected to possess a high PSNR and a low MSE value. Thus, all the denoising methods are analysed by computing PSNR and MSE.

Table 1: PSNR Analysis for Rician noise with noise 3%.

Technique	Image with Rician noise
LMME	24.2066
LM	25.4219
N3I	25.1576
Wiener	25.2032
Bilateral filter	24.3092
Adaptive non local	24.4765
Blockmatching and 3D filterig	24.8741
Volterra	**22.6525**

The PSNR is used to evaluate the quality between the denoised image and the original image. The PSNR formula is defined as follows:

$$\text{PSNR}=\frac{10\ \text{X}\log 10(255\text{X}255)/}{1/\text{HXW}\sum_{y=0}^{W-1}\sum_{x=0}^{H-1} f(x,y)-g(x,y)^2}\text{dB} \qquad (31)$$

Where 'H' and 'W' are the height and width of the image, respectively; and f(x, y) and g(x, y) are the grey levels located at coordinate (x, y) of the original and the denoised image respectively.

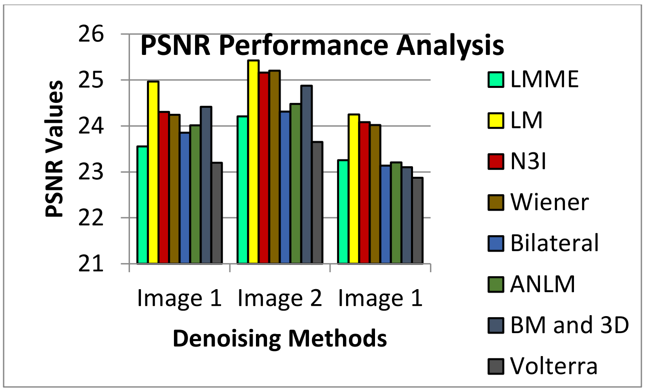


Fig: 2 PSNR Analysis

The MSE is used to evaluate the quality between the denoised image and the original image. The MSE formula is defined as follows:

$$\text{MSE}=1/\text{HXW}\sum_{y=0}^{W-1}\sum_{x=0}^{H-1} f(x,y)-g(x,y)^2 \quad (32)$$

Where 'H' and 'W' are the height and width of the image, respectively; and f(x, y) and g(x, y) are the grey levels located at coordinate (x, y) of the original and the denoised image respectively.

The corresponding graph for table 1 is presented in Fig 3.From the above table and graph, it is shown that the Volterra filter performs well than the other existing methods. It has got higher value than other method.

The corresponding graph for table 2 is presented in Fig 2.Our work outperforms all other methods with its lower value.

The plotted graphs has depicted that the Volterra shows the maximum PSNR and the minimal MSE value. LMMSE works better but since a wide range of performance gap can be observed between LMMSE and Volterra.

Table 2: MSE Analysis for Rician noise with noise 3%

Technique	Image 1	Image 2	Image 3
LMME	12.4366	12.9365	11.7161
LM	13.7537	14.2535	13.2539
N3I	15.6496	16.1493	15.1499
Wiener	14.4687	14.5683	13.9691
Bilateral filter	14.7568	15.1563	14.4513
Adaptive non local	12.5451	12.9445	11.9457
Blockmatching and 3D filterig	12.5683	13.2676	12.3069
Volterra	11.9521	12.1513	11.4529

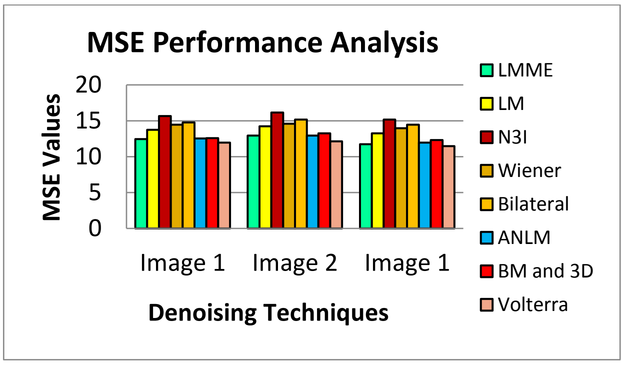


Fig:3 MSE Analysis

V CONCLUSION

In this work, several denoising filters were employed over noisy images, in order to determine the potentiality of every filter. Also, the Rician were given a trial.

Denoising filters such as Volterra, Linear Minimum Mean Squared Error, Histogram based denoising filter, Wiener Filter and Maximum Likelihood Estimation based filter were applied over the noisy image and we found that Volterra filter outperforms all the other filters in terms of quality and noise removal. Volterra is the only filter that shows the highest PSNR and the lowest MSE value. Also, performance gap can easily be observed. Thus, Volterra is the best denoising filter. So henceforth this filter can be called as VL. This filter can be applied in all type of 2D input data to get noise free data.

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